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Market Liquidity and Mutual Fund Performance during the Financial Crisis

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Abstract

Market illiquidity influences mutual fund performance differently between crisis and non-crisis period. A significant drop in market liquidity makes investors panic leading to the early and large redemption. To respond the redemption request, fund managers have to liquidate the portfolio putting pressure on the asset prices, so the underperformance of mutual fund is recognized in non-crisis period. However, the result of illiquidity is different during crisis. Market illiquidity is positively related to all fund classes. This could then be interpreted as the evidence of management skills, market-timing and volatility-timing skills in fund managers to provide superior fund performance. Moreover, the further investigation on management strategy supports the evidence of manager skills in active funds to minimize the loss during the crisis.

Keywords: Market Illiquidity, Mutual Fund Performance, Volatility, Active Fund, Passive Fund

1. Introduction

A drop in market liquidity have brought a lot of concern in the time of market distress. As mentioned in Brunnermeier and Pedersen (2009), there is a link in asset's market liquidity (i.e., the ease with which it is traded) and traders' funding liquidity (i.e., the ease with which they can obtain funding). Market liquidity can be explained in 5 features. First, it can suddenly dry up. Trader requires capital when he buy a security, so he can use security as collateral and borrow against it, but he cannot borrow for the whole price. The difference between security's price and collateral value is margin that must be financed with trader's own capital which we called funding liquidity. When funding liquidity is tight, trader becomes reluctant to take on position especially for high-margin securities, so this would reduce market liquidity. In other word, the larger margin requirement, the more restriction for trader to provide market liquidity. Finally, it leads to dry-up in market liquidity or fragility of market liquidity. Second, market liquidity has commonality across assets and asset classes. Liquidity commonality refers to the synchronicity of individual asset with aggregate market-wide liquidity movement. In other word, market liquidity and fragility co-move across assets when funding constraint affected speculators to provide market liquidity of all assets. Third, market liquidity is related to volatility in the time of market uncertainty. Liquidity shock can lead to price volatility that raise the expectation on future volatility. It caused the increase in margin constraint that lowers market liquidity eventually. Fourth, market liquidity is subject to "flight-to-quality" or "flight-to-liquidity" in other word. It arises when funding liquidity becomes shortage, so that speculators cut back on the market liquidity, mostly capital intensive, i.e., high-margin securities. Last, market liquidity is co-moves with the market since funding conditions do. Thus, market liquidity and funding liquidity are mutually reinforcing, and it might lead to liquidity spirals.

Rosch and Kaserer (2014) demonstrate a transmission channel causing market illiquidity during the market downturn which are liquidity commonality (i.e., the co-movement of an asset's liquidity and market liquidity) and flight to liquidity (i.e., the situation where investors tend to move portfolio from illiquid to liquid). Market liquidity is highly sensitive to the change in funding condition. Funding shock could bring an unfavorable margin requirement leading to an increase in the probability of margin calls. Moreover, trader might force to partially liquidate the portfolio putting pressure on asset's price and tighten funding constraint further which make market liquidity dry ups. Liquidity spiral is more likely to occur.

Overall, the severe effect of market illiquidity is from the restrictive funding liquidity that normally occur in the time of market uncertainty, it incurs more transaction cost and downward pressure in asset price. Thus, it brings more attention to study the liquidity problem that still exists in the market from the past until nowadays.

In a context of mutual fund, a severe drop in market liquidity becomes more challenging for portfolio management. Liquidity mismatch is more likely to occur that increases transaction cost and price impact for securities that mutual fund holds. The illiquidity in the market puts more pressure on asset's price downward (e.g., panic selling) causing the lower fund performance. Furthermore, the large amount of money withdrawal from the fund could bring an unsatisfied fund performance that possibly lead to the worst case called fund runs. For example, previous research study about the runs on money market mutual fund in 2008. Therefore, fund managers have to manage portfolio liquidity carefully in response to investor's transaction (e.g., redemption).

In this research, the role of market liquidity and mutual fund performance during the financial crisis is examined. In addition, market liquidity and fund performance are observed during the normal period to classify the difference of liquidity between these two periods (i.e., crisis and non-crisis period). Mutual fund is categorized according to the asset classes that mutual fund holds namely money market fund, bond fund and equity fund. Market illiquidity is also classified by fund classes namely money market illiquidity, bond market illiquidity and equity market illiquidity. In other word, the objective is to investigate the role of illiquidity in specific market on specific mutual fund.

There are two reasons that various fund classes are focused. First, Cespa and Foucault (2014) find that liquidity providers often learn information about an asset from prices of other assets. They mention that the shock specific to liquidity supply (e.g., margin constraint and fund withdrawal) in one asset class propagate to other asset classes. They show that cross-asset learning makes the liquidity of asset pairs interconnected: if the liquidity of one asset drops, its price becomes less informative for liquidity providers in another asset, and therefore the liquidity of this asset drops as well. Thus, they recommend further research to study the liquidity spillover across asset classes. To apply with mutual fund, it is essential to study on different types of mutual funds so we can see how these asset classes are interconnected. Second, several studies (Strahan and Tanyeri, 2015; Schmidt, Timmermann, and Wermers, 2016) examine runs on money market fund responses to systematic liquidity shock in the collapse of Lehman Brothers, 2008. They mention about the asset pools that subject to run-risk which are cash-like liabilities. During the crisis, investors demanded unusually high-frequency access to their cash, while the liquidity of assets plunged. Funds hardest hit by investor runs reacted initially by meeting withdrawal demand and by selling off the safest and most liquid holdings. As a result, immediately after the run ended, hard-hit funds had increased portfolio risk. The prime money market fund is the most heavily affected by a large fund outflow compared to other funds. Choi et al. (2020) examine corporate bond fund and asset fire sale in the financial crisis 2008. They detect the corporate bond market is less liquid than the equity market and that bond funds are more vulnerable to investor runs than equity funds. Corporate bond funds hold more liquid assets to cushion against redemptions. Therefore, bond funds do not have to liquidate corporate bonds in large volumes to accommodate investor redemptions. Equity funds, by contrast, hold only small liquid cushions in the form of cash. Hence, to meet redemptions, they must sell equities in large volumes, which plausibly leads to equity fire sales. We can see that the market illiquidity affects different mutual funds differently. Some funds that are more sensitive to market illiquidity (e.g., money market fund) would have more trouble in their performance, eventually it might lead to fund runs in the worst-case scenario. Some funds (e.g., equity fund) that are less sensitive to market illiquidity would recover themselves from crisis smoothly than other funds.

This study contributes to prior literature in the following several aspects. First, to the best of my knowledge, this study provides the first evidence to test mutual fund performance classified by asset class. Several studies (Paster and Stambaugh, 2003; Acharya and Pedersen 2005; Amihud and Noh, 2016) have studied the effect of liquidity risk on stock return. They find that illiquid stock has higher return than liquid stock because liquidity premium is positively priced in illiquid stock. Foran and O'Sullivan (2014) study the liquidity risk on UK equity fund. They find the strong role of stock liquidity and systematic liquidity risk in fund performance evaluation. Most of prior studies focus on the liquidity in an individual asset or a single type of fund. Thus, it would fill the literature gap to interpret liquidity in term of fund classes (e.g., money market, bond, and equity). In addition, Cespa and Foucault (2014) examine the relationship between price informativeness and liquidity that caused liquidity spillover across asset classes. Therefore, to study the liquidity effect on fund classes would give more contribution on how sensitivity of liquidity is different across funds. Furthermore, the role of illiquidity on fund performance in different periods (i.e., normal and crisis period) is investigated. Thus, the difference of market liquidity between crisis and non-crisis period is observed clearly.

Second, in this research, Asia emerging mutual funds are investigated namely China, India, Indonesia, South Korea, Taiwan, and Thailand (see MSCI definition). The reasons that Asia emerging funds are focused are the following. Many studies rely on the research of developed mutual fund (e.g., US. and Europe). Evidence on Asia emerging funds are scarce. Bekaert et al. (1998) mention emerging market has low correlation with developed market. It considered as different enough as stand-alone asset class in global portfolio management. Moreover, Ramasamy and Yeung (2003) find that the growth of emerging mutual fund has been robust compared to developed fund and it is expected to grow double-digit annually. Therefore, we can observe the increasing important role of Asia emerging mutual funds to the global financial market.

Last contribution, market illiquidity affects investment strategies of mutual fund. Several studies (Jensen, 1986; Gruber, 1996; Wermers, 2000) mention that active management funds tend to underperform passive management funds. Actively managed funds aim to earn superior returns to the market. As a result, it caused high expense and transaction cost for fund managers to beat the market. In contrast, passive funds aim to replicate market portfolio index which induce less expense and transaction cost, so the performance of passive

fund is superior relative to active fund on average. Nevertheless, the argument is opposite during the global financial crisis, most active funds tend to outperform passive funds which indicate the evidence of stock-selection skill in active management strategy (Wermers, 2000; Petajisto, 2013). In addition, Frino, Gallagher and Oetomo (2006) investigate the analysis of liquidity and information of active and passive funds. They mention that active managers convey a valuable information, thus they can add value to investor and beat the benchmark indices. Passive funds in contrast, are entirely liquidity-motivated which incurs higher liquidity cost and lower returns than active funds. To be concluded, when market becomes illiquid, it would make active funds to be more active to beat the market that possibly caused superior fund performance than passive funds that try to mimic market portfolio. Therefore, it is essential to investigate further on the role of market illiquidity on active and passive funds. Whether illiquidity influence active and passive performance differently, so this would give more contribution on investment strategies of fund managers in crisis.

To sum up, by exploring various fund classes and illiquidity measures helps to better understand the sensitivity of market illiquidity on different types of fund in crisis. It sheds further light on how market illiquidity looks like. Moreover, the investigation of management fund offers the implication of management skills in fund managers. This should be useful for institutional investors, fund managers, and risk management officer to implement investment strategies to deal with illiquidity in crisis.

The remainder of this paper is organized as follows. In section 2, the research hypotheses on each fund type are offered. This shows the expectation outcome with supporting literature reviews. Section 3, data sources, illiquidity proxies, and multi-factor models are provided. Section 4 reports the discussion of empirical results. Section 5 is the contribution on management strategy funds. Conclusions follow in Section 6.

2. Research hypotheses

H_0 : Money market fund performance is negatively related to money market illiquidity. The higher illiquidity in money market, the lower performance of money market fund.

This relationship is supported by Strahan and Tanyeri (2015) and Schmidt, Timmermann, and Wermers (2016). Money market fund is perceived to be the safest and highest liquidity compared to other asset classes (e.g., bonds and stocks). However, it suffers early withdrawal from investors during the global financial crisis. During the crisis, liquidity mismatch is occurred in money market fund. Investors demand high frequency to obtain cash that force asset sales immediately and put pressure on asset prices. Net asset value of the fund declines as investors redeem the fund in large amount. Eventually, the situation called fund runs occurred. Therefore, money market fund is expected to have poor performance when liquidity in money market falls.

H_0 : Bond fund performance is negatively related to bond market illiquidity. The higher illiquidity in bond market, the lower performance of bond fund.

During the crisis, the phenomenon called flight-to-quality is more likely to occur. It is closely related with flight-to-liquidity where investors prefer to shift from illiquid to liquid assets as they turn to be more risk-averse. Choi et al. (2020) find that bond market is less liquid than equity market and that bond funds are more vulnerable to investor runs than equity funds. Friewald, Jankowitsch, and Subrahmanyam (2012) mention that the rise in illiquidity is significantly negatively affected bond prices. Bond price declines more in speculative bond compared to investment grade bond. Therefore, bond fund is expected to have poor performance when liquidity in bond market falls.

H_0 : Equity fund performance is negatively related to equity market illiquidity. The higher illiquidity in equity market, the lower performance of equity fund.

Coval and Stafford (2017) show that equity fund is experienced an asset fire sale due to the redemption in crisis and even in normal period. Choi et al. (2020) mention that equity fund holds less cash to cushion for liquidity. To meet redemption, fund managers must sell equity in large portion leading to equity fire sales. Therefore, equity fund is expected to have poor performance when liquidity in equity market falls.

3. Data & Methodology

To measure mutual fund performance in 6 Asia emerging markets (e.g., China, India, Indonesia, South Korea, Taiwan, and Thailand), fund characteristics, fund net assets and fund returns are collected from Morningstar database. In this research, fund category is divided according to global board category in Morningstar database namely, money market fund, bond fund and equity fund. The summary statistics of open-ended funds in each country is shown in Table A. The period window is between 2004-2019 that covers both crisis and non-crisis period. The CBOE Volatility index is used to classify crisis period from normal period that collected from CBOE website. The illiquidity proxies including short-term yield volatility for money market,

long-term yield volatility for bond market and price volatility for stock market are collected from Datastream database.

Table 1 : Summary statistics of mutual fund category in 6 Asia emerging markets; China, India, Indonesia, South Korea, Taiwan, and Thailand (Unit : Million USD)

	China			India		
	Money Market Fund	Bond Fund	Equity Fund	Money Market Fund	Bond Fund	Equity Fund
No. of funds	49	70	20	178	762	619
Asset Under Management	104,757.587	13,644.627	12,412.463	379,182.510	481,634.821	278,044.312
Mean	2,137.910	192.178	620.623	2,130.239	632.067	449.910
Median	236.453	56.507	391.751	548.209	229.656	164.036
Standard Deviation	4,339.931	426.236	882.392	2,489.114	886.190	689.059
Maximum	24,592.942	2,186.098	3,673.232	9,376.030	4,091.473	3,632.426
Minimum	2.916	0.705	9.323	1.841	1.681	0.547

	Indonesia			South Korea		
	Money Market Fund	Bond Fund	Equity Fund	Money Market Fund	Bond Fund	Equity Fund
No. of funds	10	39	43	92	156	964
Asset Under Management	1,814.242	1,279.535	2,774.638	85,946.957	5,397.299	45,921.723
Mean	226.780	42.651	73.017	934.206	35.047	47.785
Median	117.147	22.121	24.055	220.084	3.203	9.481
Standard Deviation	260.910	71.144	158.974	1,456.009	108.598	140.621
Maximum	774.664	358.732	891.815	6,214.269	883.467	1,414.529
Minimum	8.766	0.013	1.757	4.175	0.006	0.002

	Taiwan			Thailand		
	Money Market Fund	Bond Fund	Equity Fund	Money Market Fund	Bond Fund	Equity Fund
No. of funds	39	18	210	31	63	185
Asset Under Management	33,088.589	1,465.805	13,366.066	15,042.956	8,022.854	18,084.582
Mean	848.425	81.434	63.648	485.257	127.347	97.754
Median	609.726	21.904	34.101	127.707	12.103	21.910
Standard Deviation	863.180	105.679	83.281	829.011	355.557	210.080
Maximum	3,027.338	390.821	484.003	3,586.893	2,258.744	1,852.413
Minimum	9.478	9.363	0.777	1.421	0.183	0.172

3.1 Market Uncertainty (Crisis)

To measure market uncertainty or crisis period, VIX index is employed in this research. VIX index is created by The Chicago Board Options Exchange (CBOE) aims to measure the 30-day expected volatility of the US stock market. In other word, it is a real-time market index that represents the market's expectation of 30-day forward-looking volatility. Derived from the price inputs of the S&P 500 index options, it provides a measure of market risk and investors' sentiments. It is also known as "Fear Gauge" or "Fear Index". In this research, the cutoff threshold of VIX is followed by Chen and Yang (2021), VIX greater than 23.81% refers to high volatility regime that associated with market uncertainty or crisis period. On the other hand, VIX below 23.81% considered as low volatility regime.



Figure 1. Historical VIX 2004-2019

Figure 1. illustrates the VIX index from 2004 to 2019. The highest volatility (around 80%) is in the end of 2008 and early 2009. Thus, in this research, the crisis periods are focused on the period, 2008 to 2009.

3.2 Market Illiquidity

Market liquidity refers to the ease with which it is traded (Brunnmerier and Pedersen, 2009). In opposite, market illiquidity means the difficulty for trading in the market. In this research, market liquidity is considered according to the mutual fund category (e.g., money market illiquidity, bond market illiquidity, and equity market illiquidity). Following Sarr and Lybek (2002), liquid market tends to exhibit five characteristics. First, *tightness* refers to low transaction cost such as difference between buy and sell prices. Second, *immediacy* represents the speed which order can be executed and the efficiency of trading, clearing and settlement system. Third, *depth* refers to the existence of abundant orders. Fourth, *breadth* means large order in volume with minimal price impact. Fifth, *resiliency* refers to the orders that flow quickly to correct order imbalance.

3.2.1 Money Market

It consists of short-term debt instruments (i.e., maturities up to one year) such as deposits, treasury bills, and commercial papers. Money market is viewed as the most liquid market with high degree of safety and low return. Based on the availability of data, the approach to measure money market illiquidity is **short-term yield volatility**. Basically, short-term yield is less volatile in the normal period, however, this relationship is vice versa during the crisis. Short-term rate is highly sensitive to the crisis and it reflects high market risk that results in inverted yield curve. Therefore, short-term volatility is employed to be illiquidity proxy for money market. Daily government benchmark bid yield is used to calculate the monthly volatility which is the standard deviation of 22-days yield.

$$Monthly\ Volatility = \sqrt{\frac{\sum_{t=1}^N (X_t - X_{average})^2}{N - 1}} \quad (1)$$

where X_t is the short-term return at time t, the frequency (t) is in monthly. $X_{average}$ is the average of 22-days return.

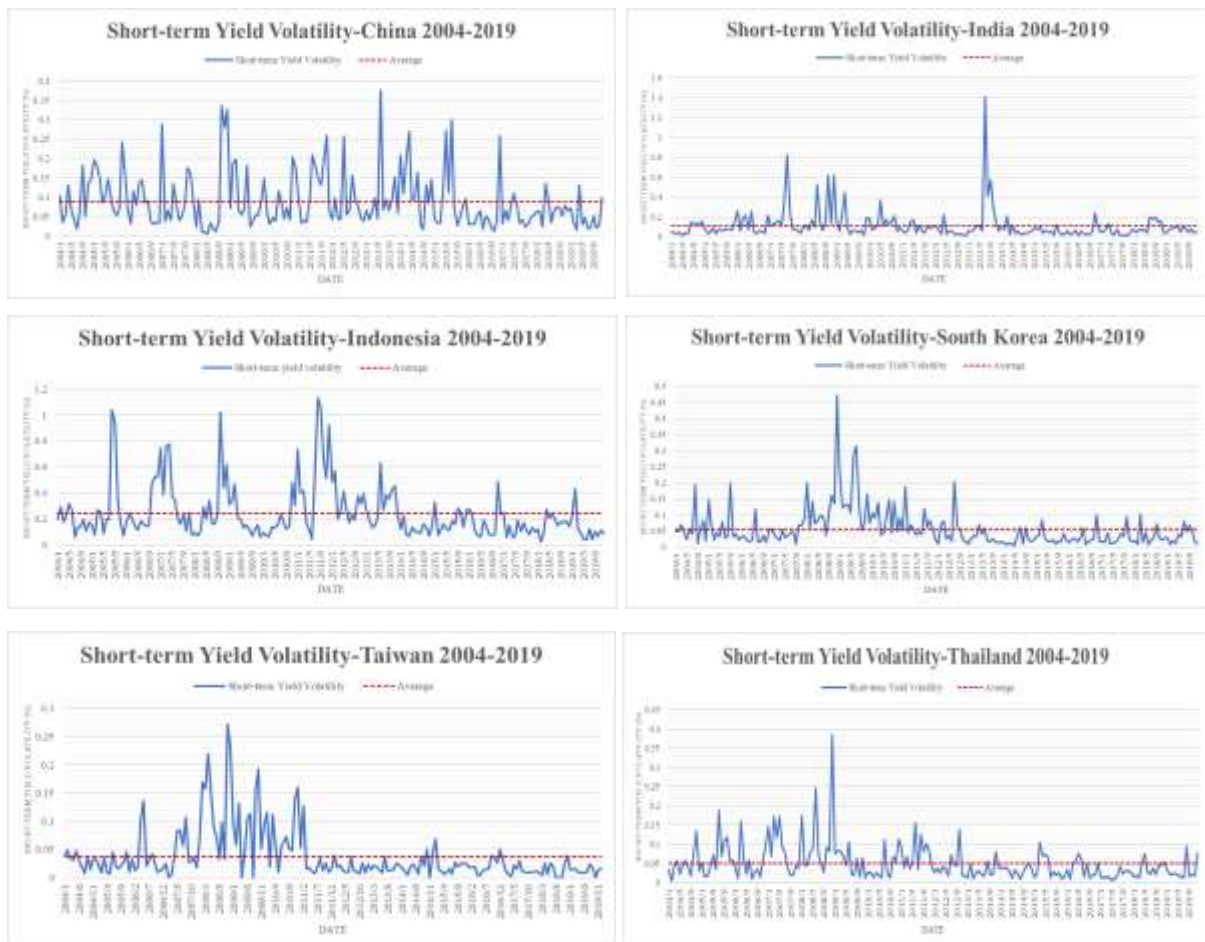


Figure 2. Historical Short-term Yield Volatility 2004-2019

Table 2 : Summary Statistics of Short-term Yield Volatility in 6 Asia emerging markets (unit: %)

	China	India	Indonesia	South Korea	Taiwan	Thailand
Mean	0.090	0.111	0.243	0.057	0.038	0.050
Median	0.066	0.068	0.174	0.038	0.022	0.037
Standard Deviation	0.071	0.148	0.206	0.059	0.045	0.047
Maximum	0.376	1.411	1.131	0.472	0.272	0.385
Minimum	0.005	0.011	0.023	0.005	0.000	0.004

3.2.2 Bond Market

Bond market consists of long-term fixed income instruments (i.e., maturities more than one year) such as government bonds and corporate bonds. Based on the availability of data, the approach to measure bond market illiquidity is **long-term yield volatility**. According to Houweling et al. (2005), they propose different proxies to measure bond market liquidity. Yield volatility is employed in this research. Yield volatility is positively related with bond spread. The higher yield volatility, the higher bid-ask spread and the lower bond market liquidity. Long-term daily government benchmark bid yield is used to calculate long-term yield volatility. All formulas are the same as money market (see 3.2.1).

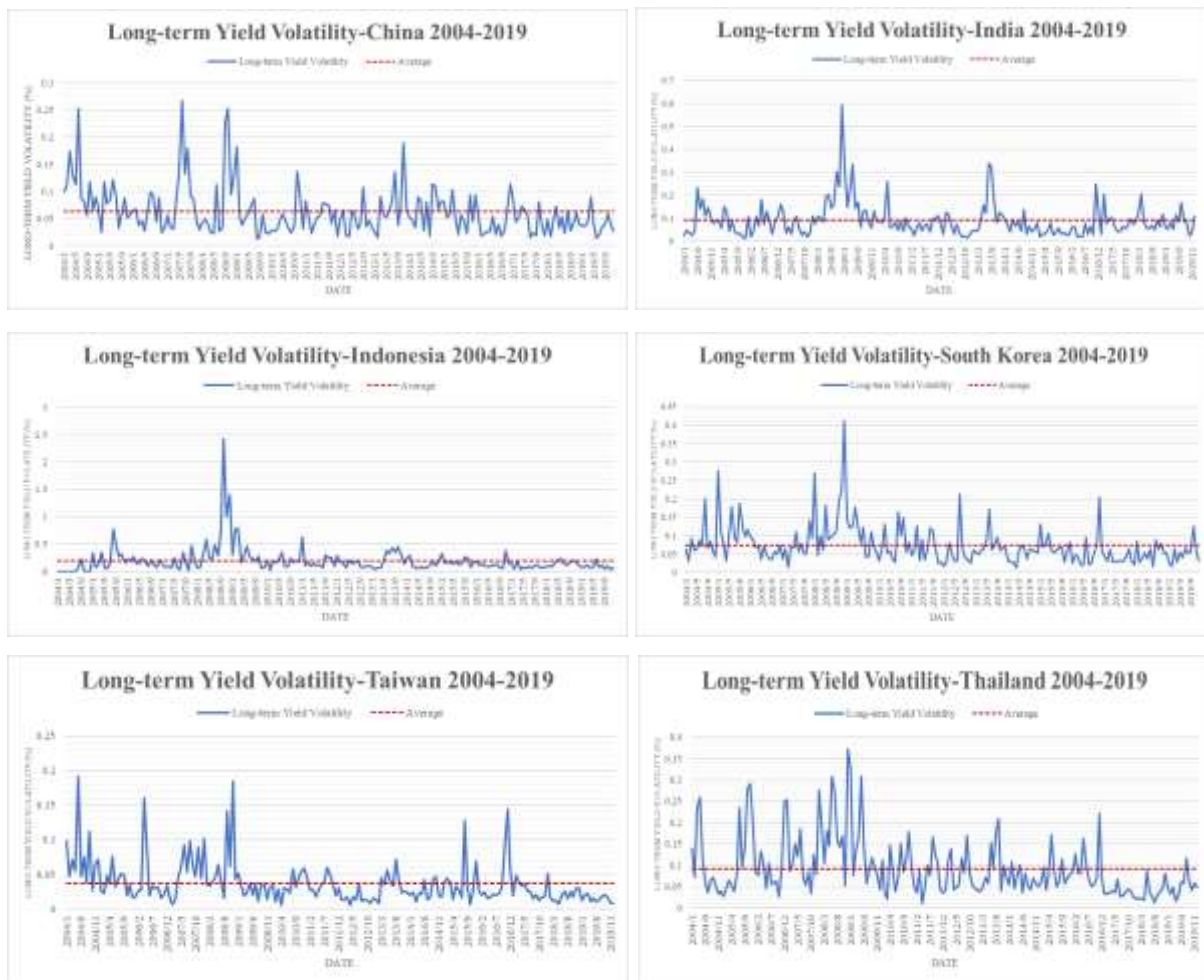
**Figure 3.** Historical Long-term Yield Volatility 2004-2019

Table 3 : Summary Statistics of Long-term Yield Volatility in 6 Asia emerging markets (unit: %)

	China	India	Indonesia	South Korea	Taiwan	Thailand
Mean	0.063	0.090	0.197	0.075	0.037	0.092
Median	0.052	0.074	0.146	0.063	0.028	0.071
Standard Deviation	0.044	0.073	0.240	0.053	0.030	0.068
Maximum	0.267	0.594	2.426	0.412	0.191	0.371
Minimum	0.013	0.012	0.000	0.014	0.005	0.010

3.2.3 Equity Market

Equity market consists of various stocks issued by company in attempt to raise the capital via different investors. There are several illiquidity proxies in equity market, so **return volatility and volume turnover** are employed in this research. First, return volatility represents the deviation of return from its average. Therefore, high return volatility, high market uncertainty thus, the illiquid equity market becomes. Price index in each stock market is used to calculate return volatility. Second, volume turnover is defined as the ratio between value of daily transaction to daily market capitalization. It measures equity market illiquidity in term of depth. In other word, turnover rate indicates the number of times that asset changes from one hand to another during a period. The reduction in volume turnover means a small portion of this market is traded which represents illiquidity in equity market. The data for volume turnover is collected from datastream database.

To be concluded, return volatility and illiquidity is positively correlated meaning that the higher return volatility, the higher equity market illiquidity. On the other hand, turnover and illiquidity is negatively correlated. The higher turnover, the lower equity market illiquidity in other word.

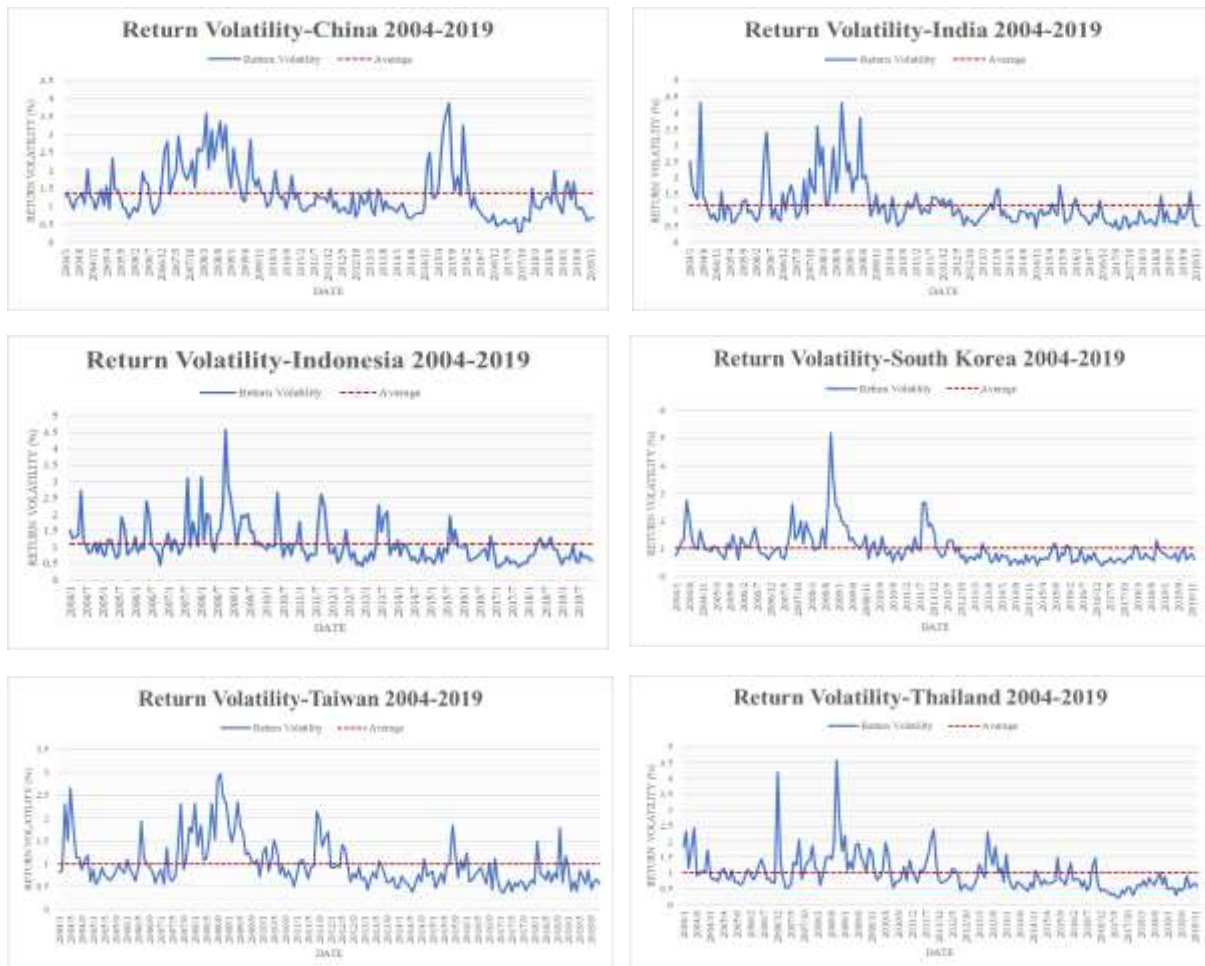
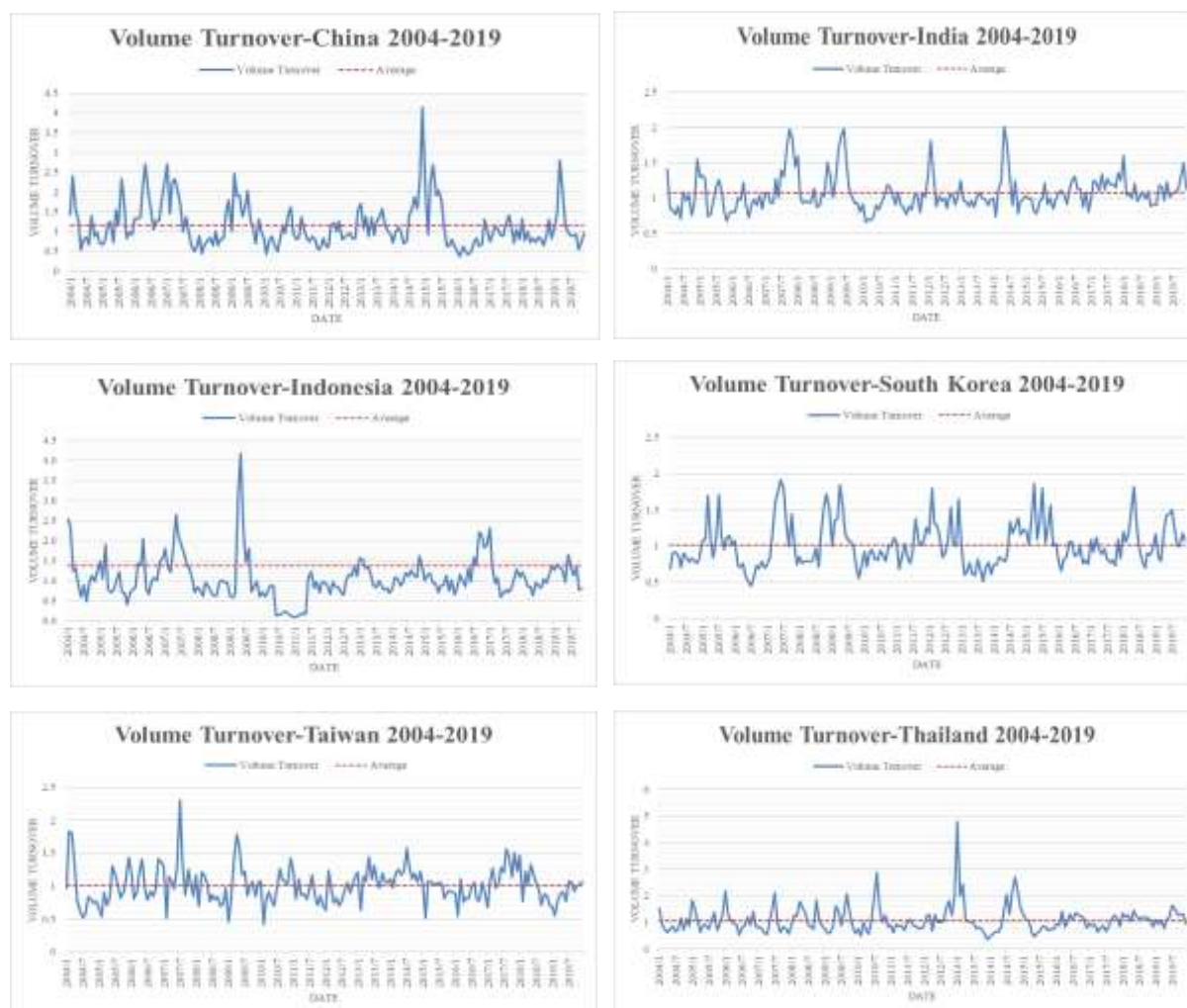


Figure 4. Historical Price Volatility 2004-2019

Table 4 : Summary Statistics of Return Volatility in 6 Asia emerging markets (unit: %)

	China	India	Indonesia	South Korea	Taiwan	Thailand
Mean	1.370	1.140	1.092	1.032	0.991	1.005
Median	1.200	0.933	0.953	0.866	0.846	0.834
Standard Deviation	0.701	0.684	0.596	0.596	0.511	0.589
Maximum	3.869	4.318	4.566	5.188	2.963	4.570
Minimum	0.280	0.383	0.371	0.387	0.354	0.221

**Figure 5.** Historical Volume Turnover ratio 2004-2019**Table 5** : Summary statistics of Volume Turnover in 6 Asia emerging markets

	China	India	Indonesia	South Korea	Taiwan	Thailand
Mean	1.150	1.073	1.381	1.017	1.011	1.091
Median	0.975	1.021	0.973	0.969	1.002	0.979
Standard Deviation	0.560	0.256	4.439	0.308	0.273	0.498
Maximum	4.133	2.017	62.121	1.911	2.298	4.777
Minimum	0.367	0.662	0.080	0.444	0.420	0.356

3.3 Mutual fund performance

There are many approaches to measure fund performance (e.g., sharpe ratio, standard deviation, and treynor ratio). The selected approach in this research is multi-factor model because the sensitivity of market illiquidity to different mutual fund categories is examined. The baseline equation of multi-factor model is expressed in Eq. (2). The interacted equation influences the differential effect of market illiquidity in times of crisis, see Eq. (3).

$$R_{i,t} - R_{f,t} = \alpha_i + \sum_{j=1}^n \beta_j * f_{j,t} + \gamma_1 ILLIQ_t + \varepsilon_{i,t} \quad (2)$$

$$R_{i,t} - R_{f,t} = \alpha_i + \sum_{j=1}^n \beta_j * f_{j,t} + \gamma_1 ILLIQ_t + \gamma_2 CRISIS_t + \gamma_3 CRISIS_t * ILLIQ_t + \varepsilon_{i,t} \quad (3)$$

where $R_{i,t}$ is the net return of fund i at month t , $R_{f,t}$ is the risk-free rate on month t . α_i is the risk-adjusted return on month t . $f_{j,t}$ is the market-specific factor j on month t . $ILLIQ_t$ is market illiquidity in non-crisis that measured by illiquidity proxies (see 3.2). $CRISIS_t$ is the dummy variables (i.e., 1 = crisis, 0 = non-crisis). $CRISIS_t * ILLIQ_t$ is the interacted variable added to the model to investigate the relationship between market illiquidity and fund performance during the crisis.

3.3.1 Money Market Fund

In this research, the money market-specific factors include level factor ($LEVEL_t$) and term factor (TS_t) from Knez, Litterman and Scheinkman (1994). These two factors represent the decomposition of yield curve shape that can be explained by Nelson and Siegel (1987)

$$r(0, T) = \alpha_1 + \alpha_2 \left[\beta \left(\frac{1 - e^{-t/\beta}}{t} \right) \right] + \alpha_3 \left[\beta \left(\frac{1 - e^{-t/\beta}}{t} \right) - e^{-t/\beta} \right]$$

where α_1 captures the level (level factor), and α_2 captures the steepness (term factor). **Level factor** represents the parallel change in the yield curve. **Term factor** measures the slope or steepness of the yield curve. It is calculated by the return difference between 10-year government bond and 1-month treasury yield. Term factor lowers treasury yield for shorter maturities and raises the yield for longer maturities.

3.3.2 Bond Fund

There are 3 factors employed in the bond model (Fama and French, 1992; Bessembinder et al., 2009; Clare et al., 2019). First, **market factor** ($R_{m,t} - R_{f,t}$) captures the market risk premium. Second, **term factor** (TS_t) or term spread captures the steepness of yield curve. It is calculated by the return difference between 10-year government bond and 1-month treasury yield. Third, **credit factor** (CS_t) or credit spread captures the reward for taking on credit risk. It is computed by the return difference between Baa rated corporate bond and Aaa rated corporate bond.

3.3.3 Equity Fund

To measure equity fund performance, Fama-French 5 factors are employed (Fama and French, 2016). **Market factor** ($R_{m,t} - R_{f,t}$) captures market risk premium. **Size factor** (SMB_t) captures the performance of small cap stock relative to large cap stock. **Value factor** (HML_t) captures the performance of value stock relative to growth stock. **Profitability factor** (RMW_t) captures the performance of robust profitability stock relative to weak profitability stock. **Investment factor** (CMA_t) captures the performance of conservative investment portfolio relative to aggressive investment portfolio.

4. Results

First, I begin the analysis by summarizing the statistics of all factors employed in the multi-factor model (see Table 6). Next, the regression analyses of market illiquidity on mutual fund classes are provided to compare the different impact of market illiquidity on fund performance during crisis and non-crisis period.

Table 6 : Summary statistics of factors used

	Money market model		Bond model			Equity Model				
	LEVEL	TS	Rm-Rf	TS	CS	Rm-Rf	SMB	HML	RMW	CMA
Mean	3.875	0.976	0.849	0.976	1.054	0.849	-0.016	0.443	0.179	0.211
Median	3.100	0.783	0.785	0.783	0.920	0.785	-0.080	0.235	0.255	0.230
Standard Deviation	2.708	0.838	5.897	0.838	0.461	5.897	1.653	1.634	1.203	1.393
Maximum	13.951	4.545	17.980	4.545	3.380	17.980	4.210	5.490	3.070	6.430
Minimum	0.008	-2.898	-27.290	-2.898	3.380	-27.290	-6.940	-3.060	-3.910	-5.860

Table 7 : The differential influence of illiquidity in crisis and non-crisis periods on money market fund

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_1 LEVEL_{i,t} + \beta_2 TS_{i,t} + \gamma_1 ILLIQ_{i,t} + \gamma_2 CRISIS_{i,t} + \gamma_3 CRISIS_{i,t} * ILLIQ_{i,t} + \varepsilon_{i,t}$$

This table reports the descriptive statistics of coefficients on underlying variables that explain the variation in money market funds in 6 Asia emergin' markets. The dependent variable is money market fund net return (Ri-Rf). The independent variables are level factor (LEVEL), term factor (TS) and money market illiquidity (ILLIQ) which is measured by short term yield volatility. The dummy variable (CRISIS) is incorporated in the model to specify the average difference in the performance of money market fund in crisis over non-crisis periods. The focused crisis is global financial crisis 2008-2009. To recognize the comparative effect of market illiquidity in two periods (i.e., crisis and non-crisis), the interacted variable (CRISIS*ILLIQ) is added to the model.

	China (No. of funds = 49)							India (No. of funds = 178)						
	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ
Mean	0.160	-0.034	0.039	0.068	0.105	-0.066	0.547	0.370	-0.167	0.063	0.042	-0.151	0.038	0.094
Median	0.187	-0.032	0.021	0.053	-0.053	-0.059	0.474	0.384	-0.179	0.076	0.042	-0.163	0.067	0.140
Standard Deviation	0.181	0.055	0.031	0.049	0.246	0.035	0.289	0.393	0.161	0.039	0.051	-0.170	0.231	0.835
Maximum	0.782	0.111	0.117	0.160	0.505	-0.014	1.389	4.627	0.104	0.231	0.226	0.127	0.627	6.477
Minimum	-0.311	-0.190	0.010	0.004	-0.223	-0.152	0.012	-31.639	-1.209	-0.001	-0.194	-1.307	-2.171	-4.105
Positive	8	49	49	23	0	49	7	177	157	27	119	128	128	128
Negative	41	0	0	26	49	0	171	1	21	151	59	50	50	50
No. of significant loadings	18	47	29	23	38	36	122	155	69	72	83	82	82	82
#Sig 1%	8	26	3	2	6	17	71	121	19	17	40	27	27	27
#Sig 5%	7	18	21	11	19	14	25	17	27	39	23	30	30	30
#Sig 10%	3	3	5	10	13	5	26	17	23	16	20	25	25	25

	Indonesia (No. of funds = 10)							South Korea (No. of funds = 92)						
	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ
Mean	0.187	0.257	-0.021	0.032	-0.055	-0.109	0.142	0.117	-0.005	0.005	-0.010	0.013	0.001	0.066
Median	0.232	0.001	0.000	0.017	0.002	0.004	-0.036	0.140	-0.013	0.004	-0.014	-0.030	-0.008	0.088
Standard Deviation	0.294	0.359	0.033	0.054	0.079	0.164	0.292	0.125	0.027	0.003	0.019	0.161	0.036	0.138
Maximum	1.035	0.853	0.006	0.147	0.018	0.068	0.671	1.946	0.108	0.022	0.084	0.728	0.134	0.372
Minimum	-0.440	-0.047	-0.087	-0.037	-0.175	-0.354	-0.163	-0.244	-0.018	-0.008	-0.033	-0.119	-0.064	-0.683
Positive	6	5	7	6	6	4	8	90	9	11	11	85	85	85
Negative	4	5	3	4	4	6	84	2	83	81	81	7	7	7
No. of significant loadings	5	5	0	5	1	0	90	84	26	7	13	11	11	11
#Sig 1%	4	2	0	4	0	0	10	10	2	1	5	1	1	1
#Sig 5%	0	2	0	1	1	0	76	72	13	4	3	5	5	5
#Sig 10%	1	1	0	0	0	0	4	2	11	2	5	5	5	5

	Taiwan (No. of funds = 39)							Thailand (No. of funds = 31)						
	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	Rp-Rf	Alpha	(β ₁) LEVEL	(β ₂) TS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ
Mean	-0.042	0.00002	0.004	-0.007	-0.042	0.007	0.042	0.058	-0.005	0.004	-0.033	-0.112	-0.011	0.289
Median	0.007	0.000	0.004	-0.007	-0.041	0.007	0.007	0.067	-0.010	0.004	-0.020	-0.042	-0.007	0.184
Standard Deviation	0.111	0.001	0.001	0.001	0.011	0.002	0.023	0.125	0.026	0.004	0.048	0.402	0.020	0.600
Maximum	0.164	0.001	0.006	-0.004	-0.020	0.011	0.122	0.606	0.091	0.008	0.000	0.160	0.023	2.872
Minimum	-0.415	-0.002	0.002	-0.008	-0.069	0.000	0.007	-1.870	-0.020	-0.018	-0.272	-2.094	-0.102	-0.631
Positive	23	39	0	0	39	39	29	2	30	1	12	5	5	5
Negative	16	0	39	39	0	0	29	1	30	19	26	6	6	6
No. of significant loadings	0	3	13	0	0	0	22	21	23	8	3	20	20	20
#Sig 1%	0	0	0	0	0	0	5	4	14	1	0	8	8	8
#Sig 5%	0	0	0	0	0	0	9	12	6	3	2	7	7	7
#Sig 10%	0	3	13	0	0	0	8	5	3	4	1	5	5	5

4.1 Money market fund

On average, alphas are negative in all countries except Indonesia and Taiwan. Negative alpha means there is no risk-adjusted fund outperformance whereas positive alpha means that fund managers are skillful to provide excess return to money market fund. The statistical significance of alpha is robust in India, South Korea, and Thailand which can explain the outperformance in money market fund by 68%, 97%, and 70% respectively. LEVEL factor represents by the short-term interest rate. All countries except Indonesia produce positive relationship between LEVEL and money market fund performance meaning that the higher short-term interest rate, the better money market fund performance. The statistical significance for LEVEL is strong in China, India, South Korea, and Thailand with number of significant funds around 95%, 87%, 91%, and 67% respectively. On average, China, India, and Indonesia shows a positive relationship between term factor and money market fund performance while the results are vice versa for the rest countries. The positive relationship indicates that term factor is positively related with fund returns during periods where yield curves are steeper. Next, money market illiquidity in non-crisis is negatively related to money market fund performance in India, Indonesia, Taiwan, and Thailand. In addition, money market illiquidity is measured by short-term yield volatility, so the higher volatility, the lower money market excess return. Crisis variable shows the average difference in money market fund performance. Money market fund performs poorly during crisis compared to non-crisis periods in China, Indonesia, and Thailand whereas the relationship is reverse for India, South Korea, and Taiwan. The last variable, CRISIS*ILLIQ that incorporated illiquidity in crisis shows the positive relationship in all countries. This could then be interpreted as the evidence of management skills. Normally, market illiquidity usually causes the difficulty to manage the fund, however, it is positively related to money market fund performance during the crisis. The positive relationship indicates that money market fund is outperformed in the time of crisis that associated with high illiquidity in the market. It implies that fund managers might somehow provide adequate liquidity inside the portfolio to absorb against the shock. In addition, it represents fund manager skills to forecast and make use of volatility, so the outperformance of money market fund might exist during the crisis. However, the sensitivity of illiquidity in crisis is small which is around 0.2% on average. It implies that there is small outperformance in money market fund. The statistical

significance is robust for China and Thailand with number of significant funds of 73% and 64% can be explained by this relationship.

4.2 Bond fund

Table 8 illustrates that all countries produce positive alpha in bond fund on average. The positive alpha can be interpreted as the management skills in fund manager to provide superior risk-adjusted return. The statistical significance of alpha is especially robust in India and Indonesia with 92% and 94% of significant funds, respectively. On average, bond funds move with the market in the same direction, but the sensitivity is so small around 0.1 on average. For term spread, the positive slope of yield curve is found in China, India, and Taiwan while the negative slope of yield curve is found in Indonesia, South Korea, and Thailand. Next, credit spread captures the reward for taking on credit risk. Credit spread is positively related to bond fund performance in China, India, Indonesia, and Thailand. Credit spread is normally reflected the economic condition. The higher credit spread indicates a concern of investors about the ability for corporate borrowers to pay back their debt. Therefore, the positive relationship between credit spread and bond fund performance implies that during the periods where investors are risk averse, bond fund returns are higher. The relationship of credit factor is reverse for South Korea and Taiwan. For ILLIQ, it is bond market illiquidity in non-crisis which is measured by long-term volatility. ILLIQ is negatively related to bond fund performance in all countries except South Korea. The negative relationship of ILLIQ indicates the underperformance of bond fund when market becomes illiquid during non-crisis period. On average, bond funds are underperformed in crisis relative to non-crisis period. However, when I incorporate illiquidity in the crisis, the result is opposite. All countries show positive coefficient of CRISIS*ILLIQ meaning that bond fund is outperformed in the crisis. This can be interpreted as the evidence of manager skills in mutual fund management. Fund managers might strategically trade on the upside volatility that existed in the crisis to gain the excess return. South Korea and Taiwan shows the high number of significant funds around 86% and 88% that can explain the positive relationship between illiquidity and bond fund in crisis.

Table 8 : The differential influence of illiquidity in crisis and non-crisis periods on bond fund

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_1(R_{m,t} - R_{f,t}) + \beta_2TS + \beta_3CS + \gamma_1ILLIQ + \gamma_2CRISIS + \gamma_3CRISIS*ILLIQ + \epsilon_{i,t}$$

This table reports the descriptive statistics of coefficients on underlying variables that explain the variation in bond funds in 6 Asia emerging markets. The dependent variable is bond fund net return (Rp-Rf). The independent variables are market factor (Rm-Rf), term factor (TS), credit factor (CS), and bond market illiquidity (ILLIQ) which measured by long term yield volatility. The dummy variable (CRISIS) is incorporated in the model to specify the average difference in the performance of bond fund over crisis and non-crisis periods. The focused crisis is global financial crisis 2008-2009. To recognize the comparative effect of market illiquidity in two periods (i.e., crisis and non-crisis), the interacted variable (CRISIS*ILLIQ) is added to the model.

	China (No. of funds = 71)									India (No. of funds = 762)								
	Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ		Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	
Mean	0.437	0.480	0.070	0.660	1.094	-1.621	-0.801	11.546		0.411	0.540	0.004	0.032	0.495	-1.722	-1.009		6.886
Median	0.338	0.471	0.066	0.591	1.117	-1.854	-0.743	11.464		0.417	0.506	-0.002	0.088	0.052	-0.949	-0.317		2.055
Standard Deviation	1.528	0.206	0.035	0.525	0.612	-4.028	0.343	5.909		1.032	0.165	0.017	0.243	1.221	1.937	1.164		8.207
Maximum	24.479	0.965	0.221	2.527	2.430	12.112	-0.185	28.726		26.465	1.114	0.127	0.583	6.114	2.935	0.414		27.104
Minimum	-12.245	-0.022	0.001	-0.110	-0.594	-10.531	-1.743	-7.823		-24.931	-0.010	-0.054	-0.851	-2.678	-8.274	-4.156		-3.963
Positive		70	71	67	68	21	0	70			761	316	535	477	87	29		729
Negative		1	0	4	3	50	71	1			1	446	227	285	675	733		33
No. of significant loadings		47	67	28	23	15	23	41			746	205	295	183	249	413		368
#Sig 1%		28	54	8	3	6	6	16			707	18	149	89	78	312		303
#Sig 5%		9	9	8	8	6	8	12			27	78	86	57	107	48		27
#Sig 10%		10	4	12	12	3	9	13			12	109	60	37	64	53		38
	Indonesia (No. of funds = 39)									South Korea (No. of funds = 156)								
	Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ		Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	
Mean	0.520	1.182	0.133	-0.345	0.861	-2.424	0.830	1.973		0.176	0.121	0.016	-0.447	-0.303	0.203	-0.469		5.689
Median	0.529	1.094	0.124	-0.293	0.634	-2.729	0.613	1.919		0.154	0.140	-0.001	-0.414	-0.096	-0.036	-0.460		6.103
Standard Deviation	2.375	0.648	0.101	0.468	1.613	2.233	0.986	2.521		0.829	0.120	0.068	0.583	0.942	1.930	0.459		4.440
Maximum	69.071	3.203	0.291	0.222	3.817	1.167	4.079	8.337		10.464	0.394	0.417	2.283	0.493	10.543	1.008		15.736
Minimum	-28.604	0.149	-0.007	-2.461	-3.674	-8.980	-0.499	-2.691		-17.160	-0.528	-0.014	-1.900	-5.681	-3.546	-1.823		-9.699
Positive		39	36	10	25	6	30	30			148	66	24	47	78	15		144
Negative		0	3	29	14	33	9	9			8	90	132	109	78	141		12
No. of significant loadings		37	34	19	5	20	12	14			111	35	113	20	38	112		135
#Sig 1%		26	31	4	1	5	4	3			67	13	77	5	11	94		113
#Sig 5%		9	2	12	3	7	4	4			31	8	15	4	12	11		11
#Sig 10%		2	1	3	1	8	4	7			13	14	21	11	15	7		11
	Taiwan (No. of funds = 18)									Thailand (No. of funds = 63)								
	Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ		Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) TS	(β ₃) CS	(γ ₁) ILLIQ	(γ ₂) CRISIS	(γ ₃) CRISIS*ILLIQ	
Mean	0.107	0.259	0.080	0.187	-1.928	-6.975	-0.524	16.519		0.107	0.093	0.014	-0.347	0.277	-0.150	-0.057		1.181
Median	0.086	0.236	0.066	0.132	-1.825	-6.876	-0.488	20.191		0.095	0.100	0.000	-0.285	0.223	-0.370	-0.032		1.093
Standard Deviation	1.338	0.133	0.094	0.407	1.977	3.161	0.449	13.529		0.612	0.084	0.050	0.430	1.068	0.902	0.191		1.709
Maximum	8.110	0.465	0.290	0.880	0.349	-0.601	0.245	29.621		7.610	0.339	0.290	0.055	2.575	2.829	0.619		9.431
Minimum	-20.303	-0.015	-0.038	-0.423	-6.660	-11.658	-1.197	-15.453		-20.380	-0.254	-0.004	-2.183	-5.205	-2.742	-1.000		-4.027
Positive		17	13	13	3	0	3	16			59	27	15	60	19	21		51
Negative		1	5	5	15	18	15	2			4	36	48	3	44	42		12
No. of significant loadings		11	15	0	13	14	8	16			50	13	37	34	27	5		33
#Sig 1%		5	13	0	11	5	4	13			41	6	29	20	8	0		8
#Sig 5%		3	2	0	1	6	3	2			7	5	7	10	11	5		15
#Sig 10%		3	0	0	1	3	1	1			2	2	1	4	8	0		10

Table 9 : The differential influence of illiquidity in crisis and non-crisis periods on equity fund

Volatility-based model : $R_{i,t} - R_{j,t} = \alpha_i + \beta_1(R_{vol,t} - R_{i,t}) + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_5 CMA + \gamma_1 VOL + \gamma_2 CRISIS + \gamma_3 CRISIS^*VOL + \epsilon_{i,t}$
 Volume-based model : $R_{i,t} - R_{j,t} = \alpha_i + \beta_1(R_{vol,t} - R_{i,t}) + \beta_2 SMB + \beta_3 HML + \beta_4 RMW + \beta_5 CMA + \gamma_1 TURN + \gamma_2 CRISIS + \gamma_3 CRISIS^*TURN + \epsilon_{i,t}$

This table reports the descriptive statistics of coefficients on underlying variables that explain the variation in equity funds in 3 Asia emerging markets. Fama-French 5 factor model is employed. The dependent variable is equity fund net return (Rp-Rr). The independent variables are market factor (Rm-Rr), size factor (SMB), value factor (HML), profitability factor (RMW), investment factor (CMA) and equity market illiquidity (ILLIQ) which measured by return volatility (VOL) and volume turnover (TURN). The dummy variable (CRISIS) is incorporated in the model to specify the average difference in the performance of equity fund over crisis and non-crisis periods. The focused crisis is global financial crisis 2008-2009. Panel A reports a multi-factor model by using return volatility (VOL) as illiquidity measurement. Panel B reports a multi-factor model by using volume turnover (TURN) as illiquidity measurement. To recognize the comparative effect of market illiquidity in two periods (i.e., crisis and non-crisis), the interacted variable (CRISIS*ILLIQ) is added to the model.

Panel A: Volatility-based model													Panel B: Volume-based model												
China (No. of funds = 20)																									
	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) VOL	(γ ₂) CRISIS	(γ ₃) CRISIS*VOL	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN					
Mean	0.886	1.102	0.566	0.222	0.257	-0.080	-0.085	-0.426	6.043	-3.059	0.886	-4.933	0.573	0.047	0.319	0.011	-0.017	-2.440	4.848	-0.919	0.818				
Median	0.941	0.987	0.580	0.166	0.335	-0.024	0.090	-0.196	7.835	-4.194	0.941	-6.164	0.582	-0.006	0.370	0.096	0.137	-3.061	5.822	-0.937	0.890				
Standard Deviation	7.502	6.651	0.056	0.314	0.261	0.190	0.312	0.569	4.950	2.511	7.502	2.438	0.047	0.268	0.274	0.224	0.327	1.715	2.189	1.107	12.005				
Maximum	34.405	2.804	0.642	0.980	0.589	0.261	0.182	0.121	13.817	1.803	34.405	0.045	0.653	0.694	0.714	0.257	0.299	0.766	6.522	2.436	11.07				
Minimum	-30.503	0.310	0.442	-0.163	-0.257	-0.620	-0.717	-1.793	-4.440	-6.583	-30.503	-7.018	-0.490	-0.452	-0.213	-0.538	-0.692	-1.112	-0.305	-1.370	1.107				
Positive	20	20	13	16	7	12	2	16	4	16	1	20	9	17	12	12	3	20	15	15	15				
Negative	0	0	7	4	13	8	18	4	4	4	19	0	11	3	8	8	17	0	5	5	5				
No. of significant loadings	1	20	3	0	2	4	2	6	13	0	17	20	1	1	2	4	0	18	0	17	0				
#Sig 1%	1	0	0	0	1	3	0	0	0	0	17	20	0	0	0	1	0	17	0	17	0				
#Sig 5%	1	0	0	0	1	1	1	2	5	0	0	0	1	0	1	2	0	1	2	0	1				
#Sig 10%	2	0	2	0	1	0	1	4	8	0	0	0	1	1	1	1	0	0	0	0	0				
India (No. of funds = 619)																									
	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) VOL	(γ ₂) CRISIS	(γ ₃) CRISIS*VOL	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN					
Mean	1.129	1.613	0.675	0.131	0.149	0.198	-0.542	-0.881	-3.589	1.970	1.129	0.965	0.660	0.044	0.062	0.255	-0.346	-5.516	-0.919	26.771					
Median	1.226	0.673	0.105	0.147	0.209	-0.564	-0.976	-3.742	2.104	1.226	0.999	0.667	0.037	0.074	0.274	-0.353	-4.574	-0.957	27.588						
Standard Deviation	6.548	1.201	0.119	0.274	0.214	0.330	0.264	1.170	3.148	1.940	6.548	0.418	0.098	0.243	0.196	0.341	0.233	2.648	0.919	12.005					
Maximum	61.856	3.720	0.935	0.937	0.715	0.851	0.916	2.422	6.256	9.681	61.856	2.397	0.883	0.773	0.531	1.084	0.823	7.993	2.662	58.994					
Minimum	-48.525	-1.650	0.146	-0.461	-0.425	-2.356	-1.483	-2.957	-15.112	-3.602	-48.525	-0.915	0.125	-0.482	-0.446	-1.164	-1.157	-12.132	-8.878	-28.450					
Positive	551	619	423	461	489	18	142	51	517	517	608	619	327	369	489	39	14	83	604	604					
Negative	68	0	196	158	130	601	477	568	102	102	11	0	292	250	130	580	605	536	15	15					
No. of significant loadings	306	619	120	17	45	258	226	266	299	299	205	619	60	3	64	77	477	7	492						
#Sig 1%	209	615	20	0	3	18	149	43	87	13	13	613	16	0	5	0	230	0	237						
#Sig 5%	49	4	49	0	8	126	63	118	157	85	6	15	0	38	26	196	0	198	198						
#Sig 10%	48	0	51	17	34	114	14	105	55	107	0	29	3	21	51	51	7	57	57						
Indonesia (No. of funds = 43)																									
	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) VOL	(γ ₂) CRISIS	(γ ₃) CRISIS*VOL	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN					
Mean	0.964	1.449	0.736	0.342	-0.268	-0.291	0.144	-0.945	2.135	-0.417	0.964	0.878	0.772	0.439	-0.117	-0.231	0.053	2.808	-1.220	-27.135					
Median	1.262	1.412	0.728	0.333	-0.231	-0.351	0.143	-0.877	2.488	-0.727	1.262	0.999	0.774	0.423	-0.186	-0.283	0.023	2.630	-1.230	-28.303					
Standard Deviation	6.205	0.932	0.081	0.176	0.155	0.293	0.213	0.371	2.014	1.636	6.205	0.464	0.086	0.192	0.141	0.266	0.244	1.220	0.961	9.672					
Maximum	41.368	2.316	0.972	0.752	0.032	0.741	0.744	-0.396	5.404	6.394	41.368	1.828	1.007	0.900	0.095	0.714	0.769	5.885	1.784	10.393					
Minimum	-41.030	-0.768	0.572	0.056	-0.780	-0.705	-0.397	-1.995	-6.015	-3.352	-41.030	-0.509	0.581	0.103	-0.716	-0.619	-0.428	0.197	-2.819	-38.924					
Positive	43	43	43	2	3	35	0	38	8	8	40	43	43	1	4	25	43	4	1	1					
Negative	0	0	0	41	40	8	43	5	35	35	3	0	0	42	39	18	0	39	42						
No. of significant loadings	18	43	12	4	2	1	5	5	5	5	18	43	28	1	1	2	17	2	24						
#Sig 1%	0	43	2	0	0	0	0	0	2	0	0	43	6	0	0	0	2	0	0						
#Sig 5%	7	0	7	1	1	1	2	2	2	2	11	0	15	1	1	1	1	8	2						
#Sig 10%	11	0	3	3	1	0	3	3	7	7	0	7	0	0	0	0	1	8	2						
South Korea (No. of funds = 964)																									
	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) VOL	(γ ₂) CRISIS	(γ ₃) CRISIS*VOL	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN					
Mean	0.339	0.640	0.673	0.089	-0.008	0.220	-0.096	-0.779	-0.244	0.497	0.339	-0.523	0.687	0.133	0.031	0.231	-0.140	0.465	0.475	-0.527					
Median	0.384	0.535	0.682	0.075	-0.025	0.290	-0.117	-0.715	-0.528	0.833	0.584	-0.497	0.687	0.126	0.020	0.321	-0.146	0.214	0.446	-0.169					
Standard Deviation	5.990	0.875	0.161	0.224	0.299	0.470	0.280	0.881	2.128	1.562	5.990	1.140	0.151	0.244	0.290	0.513	0.287	2.722	1.107	2.782					
Maximum	53.585	3.945	1.199	1.071	1.175	1.143	0.986	3.510	9.144	5.876	53.585	2.458	1.193	1.102	1.089	1.267	0.936	10.933	3.709	8.577					
Minimum	-58.872	-2.919	0.088	-0.487	-0.898	-2.048	-0.853	-4.886	-7.647	-6.548	-58.872	-3.738	-0.149	-0.486	-0.921	-2.215	-1.101	-8.820	-1.933	-9.922					
Positive	838	964	650	448	683	285	107	351	714	714	344	964	716	530	697	246	517	583	449	449					
Negative	126	0	314	516	281	679	857	613	250	250	620	0	248	434	267	718	447	381	515						
No. of significant loadings	178	954	163	145	433	72	201	128	192	192	175	957	222	147	451	96	63	176	135						
#Sig 1%	64	949	41	15	175	4	52	37	60	60	7	951	76	11	200	8	9	17	16						
#Sig 5%	75	3	52	66	162	27	82	50	64	64	84	2	82	64	175	41	26	93	46						
#Sig 10%	39	2	70	64	96	41	67	41	68	68	84	4	64	72	76	47	28	66	73						
Taiwan (No. of funds = 210)																									
	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) VOL	(γ ₂) CRISIS	(γ ₃) CRISIS*VOL	Rp-Rr	Alpha	(β ₁) Rm-Rr	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN					
Mean	0.535	1.990	0.603	0.292	-0.467	-0.095	-0.174	-1.893	0.857	0.453	0.535	-1.284	0.651	0.409	-0.374	-0.012	-0.326	1.529	3.564	-3.397					
Median	0.835	2.174	0.640	0.295	-0.522	-0.062	-0.167	-2.066	0.814	0.576	0.835	-1.253	0.668	0.417	-0.221	0.000	-0.332	1.447	4.227	-4.153					
Standard Deviation	5.520	1.135	0.146	0.311	0.306	0.232	0.278	1.068	2.260	1.617	5.520	1.736	0.152	0.358	0.257	0.206	0.301	1.828	4.891	4.468					
Maximum	31.808	4.065	0.828	0.946	0.539	0.423	0.747	1.110	9.065	7.265	31.808	2.921	0.901	1.160	0.562	0.442	0.745	5.267	12.770	11.740					
Minimum	-32.451	-1.502	0.208	-0.372	-1.074	-0.789	-1.322	-3.923	-8.807	-6.692	-32.451	-4.749	0.259	-0.338	-0.906	-0.619	-1.486	-2.931	-12.167	-13.405</					

funds are underperformed when they are subjected to the illiquidity. The relationship of illiquidity and equity fund performance is positive in volume-based model for all countries except China and India. It indicates that the higher volume turnover or the lower illiquidity, the higher equity fund returns. Even ILLIQ factor in both models shows different direction of coefficient, the interpretation is the same. Thus, it can be concluded that illiquidity causes the underperformance of equity funds in non-crisis period. Last, the role of illiquidity in crisis is augmented in the model to identify the difference of illiquidity between crisis and non-crisis periods. In volatility-based model, India, South Korea, and Taiwan represent positive relationship between illiquidity and equity fund performance. It indicates that the higher volatility leads to the outperformance of equity funds in crisis. This implies that fund managers might implement some trading strategies during the crisis. For example, fund managers might have volatility-timing, so they can use upside volatility as the rare opportunity to trade and obtain a superior performance. For volume-based model, most of the countries exhibit negative relationship. The lower turnover or the higher illiquidity, the higher equity fund returns. The interpretation of illiquidity and equity fund performance in crisis is the same in both models. This relationship is strongly supported by the evidence of fund manager skills in crisis.

5. Further investigation on fund management strategy

The performance of active and passive funds has been discussed for a decade. Actively managed funds on average show up an inferior performance and only few funds can produce the expected returns sufficient to cover their costs (Gruber, 1996; Fama and French 2010). Nevertheless, some literature (Kremnitzer, 2012; Petajisto, 2013) demonstrate the evidence of stock-picking skills and active shares holding that lead to the outperformance of active funds during the crisis. In previous section, I found that some equity funds are outstanding during the crisis, so the further investigation on mutual fund management strategy would help to identify the investment strategies that fund managers use to provide the better performance during the crisis. This research extends the existing literature to examine the performance of active and passive equity funds incorporated with the role of illiquidity to observe the sensitivity of illiquidity on each management fund.

In this section, mutual funds are classified by management strategy namely active and passive funds. Active management aims to beat the market return, in other word, a better return above the market index. In addition, active management require a significant role of portfolio management team to analyze the market by using various trading and investment strategies. In contrast, passive fund management aims to follow the return from market portfolio by replicating the market index and minimizing the tracking errors.

Table 10 : The number of active and passive management funds.

This table shows the number of equity funds categorized by the management strategy (i.e., active vs. passive) in 6 Asia emerging markets.

	Equity Fund					
	China	India	Indonesia	South Korea	Taiwan	Thailand
Threshold	0.67	0.75	0.75	0.7	0.7	0.7
Active	11	292	24	735	155	151
Passive	9	327	19	229	55	34
Total	20	619	43	964	210	185

The criteria to identify active and passive funds is focused on mutual fund beta relative to market beta. Theoretically, market beta is equal to 1, so mutual fund beta which is closed to 1 is considered as passive funds. The reason is because the objective of passive funds is to mimic market portfolio, so beta of passive funds should be close to 1. On the other hand, mutual fund beta which is far away from 1 or above 1 is indicated as active funds. Actively managed funds aim to overcome the market. They are not necessarily followed the market, so their betas should be far away from 1 or above 1. The threshold for active and passive funds in each market is determined by the average of mutual fund beta from single-factor model, so mutual fund beta above the average is considered as passive funds. The threshold for active and passive funds are demonstrated in Table 10. Mutual fund beta above the threshold is indicated as passive funds. Mutual fund beta below the threshold or more than 1 is considered as active funds.

Table 11A : The differential influence of illiquidity on equity fund management strategy.

$$R_{i,t} - R_{f,t} = \alpha + \beta_1(R_{i,t} - R_{f,t}) + \beta_2SMB_t + \beta_3HML_t + \beta_4RMW_t + \beta_5CMA_t + \gamma_1VOL_t + \gamma_2CRISIS_t + \gamma_3CRISIS_t + VOL_t$$

This table reports the volatility-based model. It compares the regression analyses of illiquidity in two different management strategies namely active and passive funds in 6 Asia emerging markets. Panel A(B) is the regression analyses of active(passive) funds.

Volatility-based model																								
Panel A : Active fund										Panel B : Passive fund														
China																								
	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS
Mean	5.350	1.178	0.535	0.258	0.286	-0.051	-0.032	-0.491	7.267	-3.601			8.796	1.010	0.604	0.178	0.222	-0.116	-0.150	-0.346	4.547	-2.397		
Median	0.964	0.770	0.555	0.185	0.361	-0.017	0.122	-0.098	8.539	-4.510			0.908	1.004	0.607	0.147	0.309	-0.116	0.059	-0.271	7.644	-4.122		
Standard Deviation	7.522	0.843	0.056	0.334	0.246	0.113	0.247	0.718	4.464	2.260			7.480	0.316	0.024	0.300	0.290	0.260	0.383	0.336	5.357	2.774		
Maximum	34.405	2.804	0.598	0.980	0.589	0.053	0.181	0.117	13.817	1.103			32.197	1.569	0.642	0.642	0.571	0.261	0.121	0.121	8.863	1.803		
Minimum	-30.505	0.310	0.442	-0.163	-0.163	-0.319	-0.454	-1.793	-2.200	-6.583			-26.051	0.486	0.566	-0.139	-0.257	-0.620	-0.717	-1.045	-4.140	-4.716		
Positive	11	11	8	9	3	7	1	10	1	10			9	9	5	7	4	5	1	6	3	3		
Negative	0	0	3	2	8	4	10	1	1	10			0	0	4	2	5	4	8	3	6	6		
No. of significant loadings	1	11	2	0	0	1	1	5	7	7			3	9	1	0	2	3	1	1	6	6		
#Sig 1%	0	11	1	0	0	0	0	1	5	7			1	9	0	0	1	3	0	0	5	4		
#Sig 5%	0	0	0	0	0	0	1	0	1	4			1	0	0	0	0	0	0	0	1	1		
#Sig 10%	1	0	1	0	0	0	1	4	3	3			1	0	1	0	1	0	0	0	0	5		
India																								
	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS
Mean	-4.108	1.521	0.610	0.126	0.084	0.166	-0.436	-0.797	-2.489	1.465			-4.471	1.695	0.733	0.135	0.207	0.227	-0.636	-0.956	-4.571	2.421		
Median	1.114	1.474	0.630	0.105	0.065	0.191	-0.464	-0.864	-2.847	1.640			1.326	2.151	0.719	0.105	0.219	0.239	-0.659	-1.371	-4.323	2.618		
Standard Deviation	6.011	1.121	0.111	0.261	0.196	0.381	0.313	1.075	3.442	1.954			6.975	1.263	0.069	0.286	0.214	0.274	0.160	1.246	2.481	1.816		
Maximum	60.155	3.613	0.919	0.806	0.649	0.851	0.916	1.950	6.256	9.681			61.856	3.720	0.935	0.937	0.715	0.824	-0.146	2.422	3.656	9.654		
Minimum	-48.525	-1.050	0.146	-0.398	-0.425	-2.356	-1.483	-2.548	-15.112	-3.602			-39.995	-1.650	0.571	-0.461	-0.280	-0.334	-1.025	-2.957	-12.426	-2.020		
Positive	267	292	197	200	233	18	69	45	228	64			284	327	226	261	256	0	73	6	289	3		
Negative	25	0	95	92	59	274	223	247	64	64			43	0	101	66	71	327	254	321	3	3		
No. of significant loadings	139	292	48	5	34	87	90	87	113	113			167	327	72	12	11	171	136	179	186	186		
#Sig 1%	86	288	13	0	3	8	69	25	33	33			123	327	7	0	0	10	80	18	54	54		
#Sig 5%	25	4	18	0	5	38	18	34	59	59			1	0	31	0	3	88	45	84	98	98		
#Sig 10%	28	0	17	5	26	41	3	28	21	21			20	0	34	12	8	73	11	77	34	34		
Indonesia																								
	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS
Mean	9.158	1.388	0.696	0.322	-0.239	-0.197	0.175	-0.928	1.799	-0.178			7.766	1.526	0.786	0.367	-0.304	-0.410	0.105	-0.965	2.559	-0.718		
Median	1.256	1.382	0.630	0.323	-0.221	-0.256	0.144	-0.864	2.394	-0.822			1.277	1.475	0.779	0.344	-0.278	-0.399	0.110	-0.920	2.690	-0.691		
Standard Deviation	5.883	0.291	0.077	0.170	0.114	0.348	0.226	0.381	2.536	2.072			6.605	0.371	0.056	0.184	0.192	0.137	0.194	0.367	0.953	0.766		
Maximum	41.368	1.875	0.972	0.724	0.003	0.741	0.744	0.485	5.404	6.394			37.778	2.316	0.904	0.752	0.032	-0.178	0.468	-0.396	4.551	0.699		
Minimum	-40.394	0.768	0.572	0.056	-0.469	-0.650	-0.211	-1.959	-6.015	-2.352			-41.030	1.000	0.714	0.058	-0.280	-0.334	-1.025	-2.957	0.965	-2.289		
Positive	24	24	24	1	3	20	0	19	5	19			19	19	19	1	0	15	0	19	3	3		
Negative	0	0	23	4	24	4	24	5	19	19			0	0	18	19	4	19	0	19	0	16		
No. of significant loadings	11	24	8	2	1	1	4	4	5	5			7	19	4	2	1	0	1	1	0	0		
#Sig 1%	0	24	1	0	0	0	0	0	2	2			0	19	1	0	0	0	0	0	0	0		
#Sig 5%	3	0	4	0	1	1	1	1	1	1			1	4	0	3	1	0	0	1	0	0		
#Sig 10%	8	0	3	2	0	0	3	2	2	2			3	0	0	1	1	0	0	0	1	0		
South Korea																								
	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS
Mean	2.043	0.679	0.634	0.123	-0.002	0.308	-0.092	-0.826	-0.117	0.489			3.847	0.515	0.798	-0.019	-0.028	-0.064	-0.111	-0.629	-0.650	0.522		
Median	0.576	0.536	0.653	0.105	-0.014	0.379	-0.098	-0.717	-0.449	0.832			0.554	0.532	0.794	-0.021	-0.065	-0.031	-0.161	-0.712	-0.878	0.842		
Standard Deviation	5.355	0.897	0.157	0.215	0.291	0.444	0.267	0.863	1.975	1.521			5.858	0.788	0.096	0.219	0.324	0.439	0.319	0.921	2.521	1.689		
Maximum	40.481	3.841	1.199	1.071	1.175	1.143	0.986	3.177	9.144	5.876			53.585	3.945	1.006	0.579	1.036	0.758	0.888	3.510	4.946	4.791		
Minimum	-58.872	-2.166	0.098	-0.487	-0.898	-2.048	-0.853	-4.094	-6.733	-6.548			-58.805	-2.919	0.488	-0.447	-0.653	-1.437	-0.720	-4.886	-6.747	-3.450		
Positive	640	758	542	353	581	215	63	271	566	66			198	229	108	95	102	70	44	80	148	80		
Negative	95	0	193	382	154	520	672	464	169	169			31	0	121	134	127	159	185	149	81	81		
No. of significant loadings	148	725	128	93	372	48	152	66	137	137			30	229	35	52	61	24	49	62	55	55		
#Sig 1%	60	720	35	11	163	0	48	21	30	30			4	229	6	4	12	4	4	16	30	30		
#Sig 5%	61	3	37	38	130	18	64	22	44	44			14	0	15	28	32	9	18	28	20	20		
#Sig 10%	27	2	56	44	79	30	40	23	63	63			12	0	14	20	17	11	27	18	5	5		
Taiwan																								
	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS	Rp-Rf	Alpha	(β1)Rm-Rf	(β2)SMB	(β3)HML	(β4)RMW	(β5)CMA	(γ1)VOL	(γ2)CRISIS	(γ3)CRISIS	(γ3)CRISIS	(γ3)CRISIS
Mean	0.062	1.957	0.560	0.245	-0.439	-0.063	-0.147	-1.870	0.596	0.557			0.059	2.081	0.723	0.424	-0.549	-0.186	-0.252	-1.956	1.590	-0.159		
Median	0.006	1.993	0.613	0.207	-0.470	-0.060	-0.138	-1.934	0.459	0.608			0.002	2.564	0.727	0.579	-0.687	-0.162	-0.245	-2.387	1.859	0.531		
Standard Deviation	0.110	1.089	0.143	0.269	0.272	0.227	0.281	1.027	2.295	1.628			0.107	1.260	0.063	0.380	0.376	0.225	0.255	1.184	2.000	1.565		
Maximum	0.149	4.065	0.776	0.809	0.188	0.423	0.747	0.733																

Table 11B : The differential influence of illiquidity on equity fund management strategy.

$$R_{i,t} - R_{j,t} = \alpha_i + \beta_1(R_{i,t} - R_{j,t}) + \beta_2SMB_t + \beta_3HML_t + \beta_4RMW_t + \beta_5CMA_t + \gamma_1TURN_t + \gamma_2CRISIS_t + \gamma_3CRISIS^*TURN_t$$

This table reports the volume-based model. It compares the regression analyses of illiquidity in two different management strategies namely active and passive funds in 6 Asia emerging markets. Panel A(B) is the regression analyses of active(passive) funds.

Volume-based model																				
Panel A : Active fund							Panel B : Passive fund													
China																				
	Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN	Rp-Rf	Alpha	(β ₁) Rm-Rf	(β ₂) SMB	(β ₃) HML	(β ₄) RMW	(β ₅) CMA	(γ ₁) TURN	(γ ₂) CRISIS	(γ ₃) CRISIS*TURN
Mean	5.350	-5.385	0.550	0.087	0.348	0.045	0.044	5.268	-2.289	0.671	5.796	-4.380	0.602	-0.001	0.284	-0.031	-0.092	4.334	-2.625	0.998
Median	0.964	-5.972	0.559	-0.001	0.409	0.112	0.162	5.794	-2.948	0.927	6.908	-6.357	0.603	-0.069	0.297	-0.015	0.127	5.850	-1.773	0.852
Standard Deviation	7.522	1.968	0.043	0.265	0.237	0.158	0.269	1.732	1.884	1.257	7.480	2.980	0.034	0.279	0.325	0.290	0.390	2.663	1.575	0.933
Maximum	34.405	-0.429	0.610	0.694	0.672	0.215	0.297	6.622	0.786	2.146	32.197	0.045	0.653	0.438	0.714	0.253	0.299	6.418	0.045	2.436
Minimum	-30.503	-7.018	0.490	-0.212	-0.009	-0.317	-0.389	0.523	-4.116	-1.370	-26.051	-6.737	0.538	-0.452	-0.213	-0.538	-0.692	0.305	-4.035	-0.773
Positive	0	11	5	10	8	17	11	2	7	7	1	9	4	7	4	5	9	1	8	
Negative	11	0	6	1	3	4	0	9	4	0	8	0	5	2	5	4	0	8	1	
No. of significant loadings	10	11	1	0	0	1	10	0	0	0	7	9	0	1	2	3	8	0	0	
#Sig 1%	10	11	0	0	0	0	10	0	0	0	7	9	0	0	1	7	0	0	0	
#Sig 5%	0	0	1	0	0	0	0	0	0	0	0	0	0	1	2	1	0	1	0	
#Sig 10%	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	
India																				
Mean	-1.108	0.944	0.602	0.055	0.013	0.224	-0.284	-0.856	-4.628	22.622	-4.471	0.983	0.712	0.034	0.106	0.284	-0.401	-0.975	-6.308	30.475
Median	1.114	0.965	0.619	0.027	-0.009	0.254	-0.314	-0.906	-5.193	24.092	1.326	1.011	0.706	0.001	0.116	0.321	-0.401	-0.991	-6.343	30.009
Standard Deviation	6.011	0.424	0.103	0.234	0.174	0.392	0.275	0.873	2.699	11.817	6.975	0.413	0.052	0.250	0.205	0.285	0.171	0.957	2.336	10.025
Maximum	60.155	1.912	0.868	0.773	0.490	1.084	0.823	2.692	7.593	46.169	61.856	2.937	0.883	0.668	0.531	0.920	0.063	1.774	-0.124	58.994
Minimum	-48.525	-0.915	0.125	-0.482	-0.363	-2.164	-1.157	-3.131	-10.877	-28.450	-39.995	-0.734	0.595	-0.423	-0.446	-0.533	-0.833	-3.878	-12.132	-0.448
Positive	287	292	162	144	230	30	36	14	280	280	321	327	165	225	259	9	47	0	324	
Negative	5	0	130	148	62	262	256	278	12	12	6	0	162	102	68	318	280	327	3	
No. of significant loadings	109	292	25	0	46	34	5	219	220	220	96	327	35	3	18	43	2	258	272	
#Sig 1%	9	286	13	0	5	0	0	96	100	100	4	327	3	0	0	0	0	0	134	
#Sig 5%	50	6	7	0	33	10	0	94	92	92	35	0	8	0	5	16	0	102	106	
#Sig 10%	50	0	5	0	8	24	5	29	28	28	57	0	24	3	13	27	2	22	29	
Indonesia																				
Mean	0.410	0.777	0.731	0.410	-0.198	-0.141	0.100	-1.073	2.793	-25.405	7.766	1.005	0.823	0.476	-0.241	-0.345	-0.008	-1.406	2.827	-29.320
Median	1.256	0.976	0.720	0.414	-0.185	-0.173	0.039	-1.130	2.527	-28.382	1.277	0.999	0.812	0.458	-0.223	-0.327	0.005	-1.614	3.021	-28.203
Standard Deviation	5.883	0.506	0.085	0.189	0.093	0.305	0.267	1.037	1.144	11.090	6.605	0.380	0.056	0.193	0.184	0.147	0.202	0.847	1.342	7.217
Maximum	41.568	1.284	1.007	0.820	-0.004	0.714	0.769	1.784	5.885	10.393	37.778	1.828	0.937	0.900	0.095	-0.087	0.423	0.136	5.402	-17.166
Minimum	-40.394	-0.509	0.801	0.103	-0.363	-0.582	-0.248	-2.635	0.900	-38.024	-41.030	0.394	0.744	0.165	-0.716	-0.619	-0.428	-2.819	0.197	-38.810
Positive	21	24	24	0	4	15	3	24	1	19	19	19	19	1	0	10	1	19	0	
Negative	3	0	0	24	20	9	21	0	23	0	0	0	18	19	9	18	0	19	0	
No. of significant loadings	11	24	15	0	1	2	2	10	14	14	7	19	13	1	0	0	0	7	10	
#Sig 1%	0	24	2	0	0	0	0	1	0	0	0	19	4	0	0	0	0	1	0	
#Sig 5%	7	0	11	0	1	1	0	5	8	8	4	0	4	1	0	0	0	0	2	
#Sig 10%	4	0	2	0	0	1	2	4	6	6	3	0	5	0	0	0	0	0	4	
South Korea																				
Mean	2.043	-0.396	0.649	0.172	0.043	0.328	-0.144	0.347	0.203	-0.217	3.847	-0.930	0.807	0.005	-0.006	-0.082	-0.129	0.887	1.304	-1.522
Median	0.576	-0.267	0.658	0.155	0.028	0.408	-0.139	0.198	-0.099	0.134	0.554	-1.027	0.792	0.001	-0.033	-0.038	-0.180	0.965	0.983	-1.237
Standard Deviation	5.355	1.106	0.148	0.232	0.275	0.481	0.282	1.035	2.597	2.774	5.858	1.156	0.085	0.240	0.330	0.491	0.305	1.227	2.774	2.573
Maximum	40.481	2.458	1.193	1.102	1.089	1.267	0.936	3.709	10.933	8.577	53.585	1.506	1.005	0.694	0.262	0.853	0.872	3.600	10.316	8.297
Minimum	-58.472	-3.335	0.149	-0.477	-0.921	-2.215	-1.101	-1.933	-7.620	-9.922	-58.805	-3.738	0.544	-0.486	-0.626	-1.659	-0.881	-1.808	-8.620	-8.459
Positive	288	735	601	426	595	175	414	352	392	56	229	115	104	102	71	169	165	57	57	
Negative	447	0	134	309	140	560	321	383	343	173	0	114	125	127	158	60	64	172	192	
No. of significant loadings	114	728	180	91	388	80	103	43	96	96	61	229	42	56	63	16	73	20	30	
#Sig 1%	4	722	64	8	182	5	6	8	14	14	3	229	12	3	18	3	11	1	2	
#Sig 5%	55	2	64	37	140	38	54	13	28	28	29	0	18	27	35	3	39	13	18	
#Sig 10%	55	4	52	46	66	37	43	22	54	54	29	0	12	26	10	10	23	6	19	
Taiwan																				
Mean	0.062	-1.017	0.607	0.356	-0.354	0.014	-0.285	1.260	3.063	-2.973	0.059	-0.037	0.773	0.558	-0.430	-0.083	-0.442	2.286	4.976	-4.591
Median	0.006	-1.094	0.655	0.310	-0.379	0.043	-0.283	1.130	4.056	-4.060	0.002	-2.379	0.783	0.751	-0.516	-0.069	-0.440	2.782	5.442	-5.313
Standard Deviation	0.110	1.707	0.151	0.214	0.225	0.205	0.300	1.781	5.060	4.588	0.107	1.605	0.067	0.450	0.328	0.194	0.276	1.762	4.103	3.905
Maximum	0.149	2.921	0.843	0.980	0.254	0.442	0.745	4.990	12.770	11.462	0.164	1.883	0.901	1.160	0.562	0.279	0.389	5.267	11.549	10.277
Minimum	-0.415	-4.749	0.259	-0.338	-0.906	-0.527	-1.486	-2.931	-12.167	-13.405	-0.338	-4.606	0.582	-0.268	-0.884	-0.619	-1.109	-2.438	-10.668	-11.062
Positive	45	155	135	11	87	22	113	119	38	38	8	55	47	7	18	3	47	51	4	
Negative	110	0	20	144	68	133	42	36	117	117	47	0	8	48	37	52	8	4	51	
No. of significant loadings	49	155	85	85	1	37	62	32	73	73	32	55	41	40	3	24	35	28	24	
#Sig 1%	12	155	62	15	0	8	24	16	10	10	6	55	37	9	0	3	15	5	5	
#Sig 5%	27	0	15	45	0	16	25	40	38	38	17	0	3	21	2	6	17	12	11	
#Sig 10%	10	0	8	25	1	13	13	16	25	25	9	0	1	10	1	15	3	11	8	
Thailand																				
Mean	-8.550	-1.490	0.641	0.244	-0.136	0.019	0.093	1.564	1.726	-1.667	-9.423	-0.654	0.738	0.073	-0.106	-0.067	-0.132	1.726	1.073	-1.153
Median	-1.637	0.657	0.247	-0.153	0.030	0.089	1.682	1.952	-1.762	-1.762	0.859	-0.640	0.739	0.149	-0.188	-0.092	-0.006	1.150	1.731	-1.881
Standard Deviation	5.084	0.699	0.075	0.100	0.163	0.221	0.669	1.226	0.976	0.976	5.440	0.723	0.095	0.211	0.231	0.228	0.275	0.783	2.008	1.837
Maximum	22.577	0.878	0.867	0.6																

the countries. However, there is an evidence that active funds can minimize the downside risk of liquidity in Indonesia, South Korea, and Taiwan. Both active and passive funds have negative exposure to the illiquidity in these countries, however, the sensitivity of active funds is smaller than passive funds. This can be interpreted in two ways. First, active funds usually do not follow the market index, so the performance of active funds are better than passive funds in the crisis. Second, it can be interpreted as the evidence of management skills in fund managers to minimize the loss of active funds.

To conclude the different effect of illiquidity on active and passive funds. I investigate further on the mean-difference test to see whether the difference between active and passive funds are significant or not.

Table 12 : Statistical test for mean difference

This table reports the hypothesis testing for the mean of CRISIS*ILLIQ in active and passive funds in 6 Asia emerging markets.

Panel A : Volatility-based model						
	China	India	Indonesia	South Korea	Taiwan	Thailand
t-stat	-1.048	-6.282	1.178	-0.262	1.603	-0.467
df	8	291	18	228	54	33
p value	0.320	0.0001	0.250	0.790	0.110	0.640
Panel B : Volume-based model						
	China	India	Indonesia	South Korea	Taiwan	Thailand
t-stat	-0.667	-8.552	1.396	6.578	2.517	-1.581
df	8	291	18	228	54	33
p value	0.520	0.0001	0.170	0.0001	0.015	0.120

The null hypothesis is that mean of CRISIS*ILLIQ for active and passive funds are equal while the alternative hypothesis is vice versa. The critical value is 5% or 0.05. According to Table 12, the volatility-based model shows that I can reject the null hypothesis in India meaning that means of active and passive funds are different from each other. For other countries, I cannot reject the null hypothesis. There is no enough evidence to conclude that they are significantly different. In volume-based model, means of active and passive funds are different in India, South Korea, and Taiwan.

The implication of mean difference hypothesis suggests that illiquidity might affect active and passive funds differently in crisis. When market declines, passive funds that has investment policy to follow market index are suffer more from the price impact and liquidity cost that leads to the inferior fund performance (Frino, Gallagher, and Oetomo, 2006). Active funds perhaps suffer less because fund managers can forecast the market to trade the securities strategically that would result in better fund performance on average (Kremnitzer, 2012). In contrary, illiquidity might not affect active and passive funds explicitly because both funds are pressured from the asset price downward and market downturn situation. Moreover, active funds are subjected to transaction cost that is especially high in the time of crisis. Therefore, there is no clear difference for the effect of illiquidity on active and passive funds.

6. Conclusions

Market illiquidity plays an important role in mutual fund management. Fund managers have to actively manage portfolio liquidity to maintain fund performance and meet the redemption demand from investors. During non-crisis period, the negative relationship between illiquidity and fund performance is existed among three fund classes. It implies that fund managers cannot provide better return when the market becomes illiquid because they suffer more from the price impact that finally leads to the asset fire sales and asset price downward. Therefore, the result of illiquidity in non-crisis is consistent with the hypothesis that illiquidity and fund performance is negatively related. Nevertheless, the effect of illiquidity is different in crisis period. The positive relationship between illiquidity and fund performance is found in three fund classes. Money market fund has small sensitivity to the illiquidity around 0.2% on average. Bond fund shows higher sensitivity to the illiquidity around 7.3% on average. The direction of illiquidity and equity fund performance is different according to the illiquidity proxy. Volatility-based model shows the positive coefficient while volume-based model shows the negative coefficient. However, the relationship is the same. Equity funds exhibit the outperformance during crisis which is associated with high illiquidity. This could then be interpreted as the evidence of management skills in fund manager to provide better fund performance. Fund managers are skillful to implement investment strategies to trade in the market. They have the right market timing skills and make use

of upside volatility as the opportunity to gain the excess return to the mutual fund. The result is consistent with the existing literature that mention about the existence of market timing skills on the part of fund managers. They exhibit superior timing ability and performance (Kon, 1983; Cheng-Few and Shafiqur, 1990; Nicolas, Bollen and Busse, 2001). Moreover, the volatility-based model is supported by the volatility timing literature. Volatility timing in mutual fund is an important factor that determines mutual fund performance and has led to higher risk-adjusted returns (Busse, 1999; Giambona and Golec, 2008). The outstanding fund performance in the crisis leads to the further investigation on management strategy in crisis to strongly support the evidence of fund manager skills. On average active funds are underperformed passive funds due to the transaction cost that is even higher during the crisis. However, the result shows that active funds have smaller sensitivity to the illiquidity compared to passive funds. It implies that active fund management has ability to minimize the loss during the crisis. Prior literature mention that funds with forecasting skills are associated with active management strategy (Lee and Rahman, 1990). Moreover, there is a noticeable performance of market timing ability between the best and worst performing funds in the crisis periods (Andreu, Matallín-Sáez, and Sarto, 2018). Thus, the further investigation of active funds is strengthening the evidence of fund manager skills to reduce the negative exposure to the illiquidity during the crisis.

Overall, these results may be useful for mutual fund investors to realize the different effect of illiquidity in crisis. This would give an implication for fund managers to strategically use illiquidity as the opportunity to obtain the higher risk-adjusted returns in mutual fund.

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The Impact of Media Platform in Creating Ecosystems for Entertainment Business

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Abstract

Information and communications technologies have facilitated the emergence of companies like Google, Amazon, Facebook, Apple or Microsoft (GAFAM), which represent business ecosystems. They compete amongst themselves and also with classical communications and entertainment groups. At the same time, their capacity to manage big data makes unlimited growth possible, towards any activity that involves large quantities of data these companies are characterized within their respective markets, this article study a complete analysis of the internet giants Google, Amazon, Facebook, Apple and Microsoft (GAFAM), which allows us to assess their impact on the media.

Keywords: GAFAM, entertainment group, big data

Introduction

Online media platforms have continued to grow over the last two decades. The major players of the online media include Google, Amazon, Facebook, Apple, and Microsoft. These leading media houses are popularly known as the GAFAM (Florida, 2012). These are companies that have a core business that is complemented by other, often apparently unrelated activities that are in fact closely linked through big data.

Table 1. What is GAFAM



Source: Author

Of all these media channels, Google is the most prolific. It is a global library that offers answers to every question that human being has. The GAFAM online media houses have overtaken the mainstream media houses by taking all the duties expected of them. For example, advertising over the Google or Facebook would reach more globally than advertising through media houses, such as the BBC and CNN among others.

GAFAM media plays a major role in the digital economy. Digital economy entails conducting businesses through the Internet. These functions include online advertisements and the sale of goods and services. The GAFAM have a global coverage and plays these

functions at a global level. In the past, mainstream media houses, including radio, television, and newspapers, controlled most of the advertisements and had the largest share of income from this service. Today, the GAFAM shares a majority of online advertisements and sales. As Internet infrastructure continues to expand, so is the online global society. This expansion creates a conducive ground for the growth and development of GAFAM industries.

Secondly, the GAFAM media has great control in the transmission of information. Expanding online society rarely seeks answers from mainstream media houses. Rather, they rely on online media houses, such as the Google and Facebook. For example, students find it easier to use online libraries and databases when conducting research rather than visiting physical university libraries. This is because of the portability nature and the ease of access that characterizes online database (Panzarino, 2018). It is also easier for companies to inform their customers about changes that occur in their administration, production and distribution of goods through social media than broadcasting through television, radio and newspapers. Companies and websites where customers come in and search for the information they need. Mainstream media houses also use Facebook to make live broadcasts so that they can reach more people rather than relying on satellite transmission.

Lastly, GAFAM technologies play important role in communication. Communication is the art of sending and receiving information. GAFAM has improved communication globally. These companies are utilized by the online global community interchangeably to promote communication between themselves (Sharma, 2018). The use of telephone calls has greatly reduced as online audio calls have become cheaper, especially when making international calls. Also, video-calls have become a major tool in communication as the GAFAM companies have improvised ways to make these calls cheaper.

The question in everyone's mind is what impact does the GAFAM have in media entertainment? It is important to note the role of GAFAM in the entertainment industry. The GAFAM has greatly impacted the players in media entertainment. In the past, it required a lot of work for a musician and visual artists to get noticed around their village and globally (Panzarino, 2018). Today, an artist requires to have an online account with some of these media to get noticed around the world. Therefore, they get to widen their territories as well as revenue from their work within a short time. The GAFAM offers several platforms through which the entertainment companies can collect music from around the world and sell them to the global society. A musician only requires to sign up with the platforms and submit an original piece of art. The company will make online advertisements, and viewers would see the product and buy it through online transactions. After that, the company will remit revenue to the artist according to the agreement between them.

Characteristics of GAFAM Companies

Internet economic giants compete in a unique way. They all depend on the Internet to sell their products. Although, they are interdependent in the market each of them uses specific way to reach their clients. One of the examples of mutual interdependency can be traced to how to google work. It is possible to access the services of Facebook, Amazon, and Apple just by creating an account in Google. However, Google can advertise its products through Facebook and sell them through Amazon (Ourand & Smith, 2018). This implies that all these giant online companies are interdependent.

Also, these online companies have unique ways through which they make their services attractive to the buyers. For example, the Apple Company produces iPhones that do not support applications from Google. On the other hand, the two companies offer YouTube services to their customers (Resnikoff, 2018). This means that the YouTube Company has to develop an application is only supported by Google and another unique application that is

only supported by iPhone. Amazon, on the other hand, offers unique entertainment services on premium terms. These services can be offered through google or Apple companies.

Expansive Growth

The GAFAM companies are characterized by their expansive growth. As the world continues expanding the internet coverage, these companies are coming up with other unique services to offer to the expanding markets. They are, therefore, characteristics by their continued acquisition of the upcoming online platforms. These companies make deals with other upcoming platforms so that they can sell their services under these giants. The upcoming online enterprises do not have the economic power to compete with the GAFAM (Mosco, 2018). Therefore, they readily accept to work under their conditions. For example, the WhatsApp application has been growing for over the last few years. Facebook has seen the potential that WhatsApp has and has acquired it. In fact, the new WhatsApp messenger is rebranded with a “From Facebook” icon to show the world that it is now a Facebook property.

Table 2 Financials of GAFAM in 2017

	Google	Apple	Facebook	Amazon	Microsoft
Stock market value (M\$)	685,730	810,000	443,700	483,000	559,000
Sales (M\$)	90,272	216,000	28,000	136,000	85,320
Origin of sales	Advertising 88% Others 21%	iPhone 61% Services 9% Others 7% iPad 7% Mac10%	Advertising 93% Other 7%	Products 72% Media 18% Cloud 9% Other 1%	Windows 9% Office 28% Server/Azure 22% Xbox 11% Advertising 7% Others 23 %
Revenue from outside the USA	57%	70%	54%	38%	54%
Employees	72,000	116,000	18,770	341,000	114,000
Investment in R&D (M\$)	16,680	11,680	7,880	22,680	12,380
Market value/ sales	7.60	3.75	15.80	3.55	6.55
Revenue/ employee (M\$)	1.25	1.86	1.49	0.39	0.75
Gross income	21%	21%	36%	22%	20%

Source: Juan Carlos Miguel de Bustos,2019

Heading into early 2020, the Nasdaq 100 index was rising. GAFAM stocks were also rising. Facebook was the first one to break lower, peaking in late January 2020. The other GAFAM stocks and the Nasdaq 100 continued to advance. Also, these giant online platforms have the biggest shares in the leading stock markets. In terms of Million Dollars’ worth, Google has 685,730; Apple has 810,000, Facebook 443,700, amazon 48,3000, and Microsoft 559,000. These are huge amounts of money invested in the stock market operations. Other upcoming companies would find it difficult to compete with these technological companies.

Leading in Stock Market Capitalization

GAFAM are characterized by their huge shares in the stoke market. Their huge shares are attributed to their market share and their income. Google and Facebook mainly depend on the advertisement. 90% of Google’s income comes from advertisement, while Facebook revenue from advertisements accounts for 71% of their total income (Molla, 2018). On the

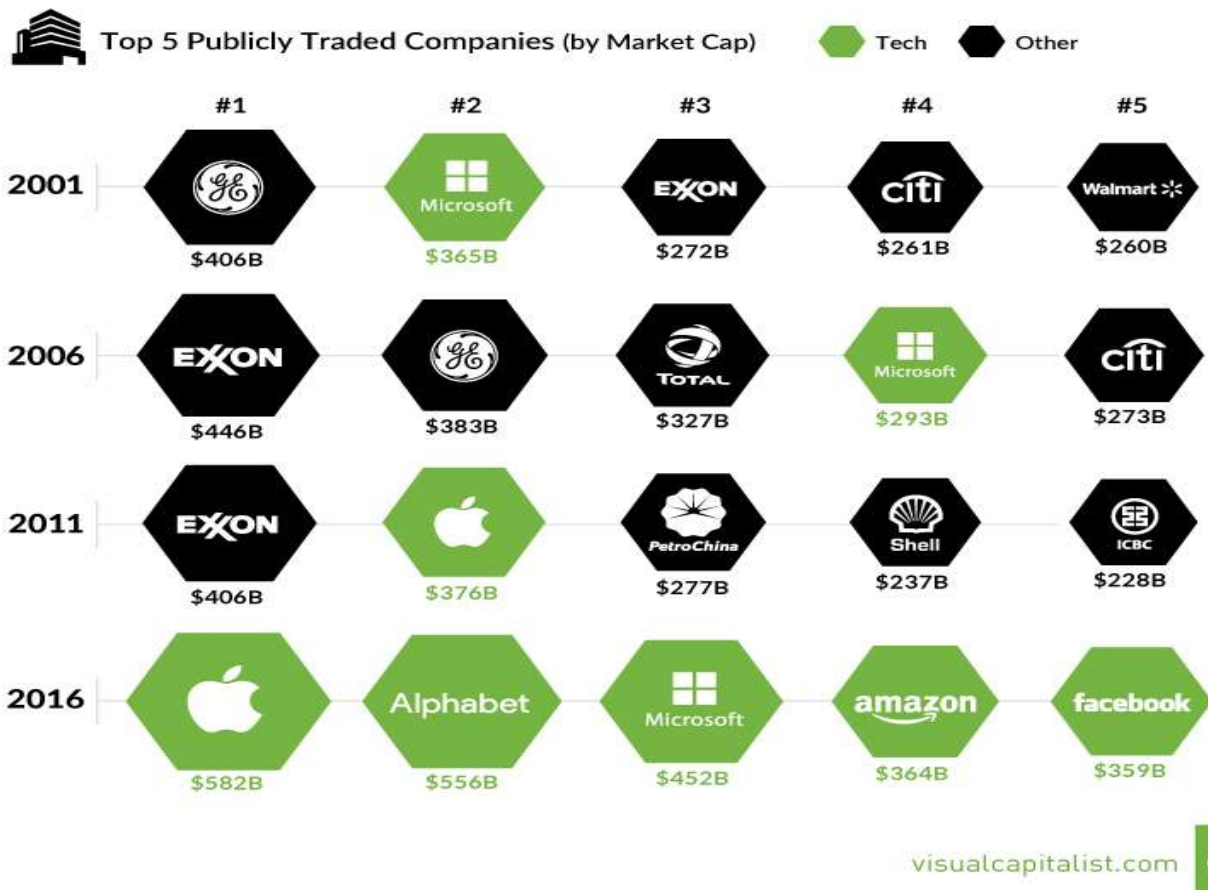
other hand, Microsoft, Apple, and Amazon do not have huge incomes from advertisements. Their revenues are accrued from the sale of different products.

Top five American corporations based on market capitalization

Chart of the Week

THE LARGEST COMPANIES BY MARKET CAP

The oil barons have been replaced by the whiz kids of Silicon Valley



Source: Visualcapitalist.com

In terms of revenue/employee ratio, Apple has the highest ratio followed by Google then Facebook. This implies that Apple has a better output per employee, when compared with the others. These statistics implies that the GAFAM companies are good at accumulating capital from their workers. Therefore, they offer their shares to the stock market to improve their market performance and continue accumulating new markets globally.

Dependence on Innovation

The GAFAM companies are good at tapping talent. These companies help in promoting individual online businesses by ensuring that their work is protected from copyrights. They do so by securing small business data and downgrading other companies trying to copy what has already been done by others (Zuboff, 2016). This way, the GAFAM ensures that individual talents are safeguarded. Also, they encourage individuals to come up with unique brands that can be supported through their platforms. Only a small share of the total amount is used to pay these companies for the services rendered. Perhaps the most

interesting characteristic that makes the GAFAM unique is their dependency on the Internet. According to Zaryouni (2015), these companies deliver their services through the use of the internet. Also, these companies depend on the internet to reach out to their customers. Of all these companies, only Apple does not fully depend on the Internet as users can have access to any information when a device is offline.

It is indisputable that GAFAM is the leading company in the telecommunication network. Other companies in the telecommunication field come second to the GAFAM. As the world is growing, the use of online telecommunication channels is growing. People are slowly moving out of the terrestrial communication. Whether a person is using Google, Amazon, Facebook, Apple, or Microsoft, they find it easier, efficient, and effective to communicate through these channels rather than the use of the terrestrial communication channels (Regina, 2012). Every ecosystem that is associated with each of the GAFAM is characterized by big data that makes each of them unique in the way they deliver their services. Their big data is associated with the company's customer and client base. Their clients compete among themselves through their platforms.

Business Models of GAFAM

Each of the GAFAM media deals with a specific business model. A business model is rational that tells how a company creates, delivers, and captures in the economic, social, and cultural contexts. Looking at the business model of the five companies, we can say that they derive their economic value from advertisements and subscriptions (Walton, 2012).

Google and Facebook specialize in advertisements. This means that their customers identify them as the best advertisement platforms. Google revenues from advertisements stand at 85% of the total revenue, while Facebook accounts for 95%. When advertising a product through Google and Facebook, you are guaranteed to reaching customers from around the world or your desired geographic area. Also, Facebook and Google allow their customers to target a specific group that constitutes their consumer base. For example, when advertising through Facebook, you can choose about the age, geographical area, and the occupation of your target customers. This implies that the two platforms have the best search for the target customers.

On the other hand, Amazon, Apple, and Microsoft derive most of their revenues from sales and subscription. This means that sales of products characterize their business model. For example, Apple, Amazon, and Microsoft revenues on sales stand at 96%, 99%, and 93% respectively. Some of the products sold by these companies include software, entertainment, and a different range of other products.

Another aspect of the GAFAM is that they depend on the electronic economy. This means that all their transactions are done on an online platform through electronic money. Although we have dangers of cyber-crimes, these companies ensure that the customers' transactions are well secured to avoid them losing their hard-earned cash to the fraudsters.

However, the GAFAM companies do practice a little of other services. For example, apart from advertisement services, google offers premium services for the sale of different products, though at a small quantity. On the other hand, Amazon, Apple, and Microsoft do little advertisements though still on a small scale (Wang, 2018). This form of diversification may grow in the future, given the level of innovation that these companies engage in.

Ecosystem, Expansion, and Competition

GAFAM companies specialize in a specific ecosystem. For example, the Apple Company sells both the software and the hardware products as a single product. Once purchased, the consumers enter into an ecosystem that forces them to consume only services from Apple Inc. (Weprin, 2018). Leaving this ecosystem means that the consumer would pay

a huge cost than the initial cost of purchasing the product. This notion of ecosystems is similar to all the other companies in the GAFAM category.

It is no doubt that GAFAM companies work in a quasi-monopolistic market structure. Their level of capital and their market level make it hard for the new entries to thrive. These companies have been expanding year after the other. Consumers of their products only require to have access to the Internet connection (Rovell, 2018). Given the level and the pace through which the world is becoming connected through the internet, translates to the rate at which these companies are expanding. For example, Google has become one of the most used search engines around the world. Even though entrepreneurs submit their products to other search engines, they do so as a second option to Google. The millions of users make people trust their services in the development of their businesses

Lastly, GAFAM companies rarely experience a lot of competition in the market. They have taken control of their market. Other upcoming online industries get consumed by these giants as they offer a better market for the services produced. However, there exists some form of competition from other players. For example, Bing is a fast-growing search engine that is competing with Google. Having joined up with Yahoo, Bing will be a serious threat to the development of Google.

Conclusion

The GAFAM media entertainment group has a lot of impact on the global economy. In the present time, it is difficult for humanity to survive without consuming a product from the GAFAM group. The development of Internet services and the limited time that people spend in their houses are some of the aspects that are making the global society to migrate to these online platforms. They have to catch up with the latest news and trends, advertise their products, and also enjoy some form of entertainment. They will choose to go for the services of the GAFAM media. With this kind of development, we expect the GAFAM to continue expanding in the years to come. Also, we expect to have other players joining up to this group in the future.

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Euclid's Elements Theorem Proving Using Isabelle

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Abstract

A lot of software has been developed for proving mathematical theorems. Research into such software is called automated theorem proving. Using the theorem proving software program Isabelle, we proved Theorems 1 to 48. When proving a theorem of Euclid's Elements using Isabelle/HOL, it is necessary to define the axioms, type specifications, etc., in a form that can be handled by the software. Through this activity, we aim to further promote the possibility of automated theorem proving.

Keywords: Automated theorem proving, Isabelle/HOL, Euclid's Elements

Introduction

Automated theorem proving (ATP) is a subfield of automated reasoning and mathematical logic, focusing on the proving of mathematical theorems using computer programs. We can use the Isabelle/HOL software [1] for mathematical theory exploration. Computer assisted proofs, involving language constructs from set theory, are a key component of theorem proving. The Isabelle is primarily intended as an educational tool for supporting theorem proving in set theory at the undergraduate level. The Isabelle allows mathematical knowledge to be organized as hierarchies of interdependent theories.

By condensing this knowledge, we can investigate properties of newly defined mathematical entities.

Euclid's Elements

Euclid's Elements is a mathematical treatise consisting of 13 books attributed to the ancient Greek mathematician Euclid in Alexandria. It is a collection of definitions, postulates, propositions (theorems and constructions), and mathematical proofs of the propositions. The books cover plane and solid Euclidean geometry, elementary number theory, and incommensurable lines. Elements is the oldest extant large-scale deductive treatment of mathematics. It has proven instrumental in the development of logic and modern science, and its logical rigor was not surpassed until the 19th century [2].

Implementation in Isabelle/HOL

When proving a theorem of Euclid's *Elements* using Isabelle/HOL, it is necessary to define the axioms, type specifications, etc., in a form that can be handled by the software.

Type declaration

The type declarations for points, line segments, and circles are as follows.

```
datatype point = "char"
datatype segment = Se "point" "point"
datatype circle = Ci "point" "point"
```

A point is represented by one character, a line segment is represented by two points, and a circle is represented by a center point and one point on the circumference.

Function implementation

Express in the form of a function that the type declared earlier is a type with the property of a line segment or a circle. First, the notation of the line segment function on Isabelle is shown.

```
locale dist =
fixes ldist :: "segment => segment => bool"
(infixl "[@]" 50)
assumes dist ref [simp,intro] : "s1 [@] s1"
and dist rev1 : "[[s1 [@] s2]] ==> s2 [@] s1"
and dist rev3 : "[[(Se x1 y1) [@] (Se x2 y2)]]
==> (Se x1 y1) [@] (Se y2 x2)"
```

The first locale statement indicates the name of the function, and second fixes statement represents the line segment AB and the line segment CD are equal as AB [@] CD. The properties of the line segment are defined in the subsequent statements.

- line segment AB = line segment AB
- line segment AB = line segment BA
- If line segment AB = line segment CD then line segment AB = line segment DC

Similarly, the function for the circle is as follows.


```

locale circledef = dist
fixes lcircle :: "point => circle => bool"
(infixl "[on]" 50)
assumes circle dist1 : "[[p [on] (Ci c r)]]
=> (Se c r) [@] (Se c p)"
and circle dist3 :
"[p [on] (Ci c r); p [on] (Ci r c)]
=> (Se p c) [@] (Se p r)"
    
```

This defines the property that the specified point is a point on the circumference of the specified circle. "A [on] CR" means "there is a point A on the circumference of a circle drawn with center C and radius R".

- If A [on] CR then line segment CR = line segment CA
- If A [on] CR and A [on] RC then line segment AC = line segment AR

In addition to this, we prepared declarations such as angles, triangles, and quadrangles as types, and functions such as right angles, parallelism, magnitude comparison, and area calculation as properties.

Proof of propositions

The process of proving each proposition of Euclid’s Elements Volume 1 (hereinafter, Book 1) using Isabelle is shown.

Here, 48 propositions are classified into five categories according to the proof target construction, triangular property, straight line property, parallel property, and area and the proof process of a proposition selected one from each category is shown.

Proposition 1_1 (Construction): Creating an isosceles triangle on a given finite straight line (line segment).

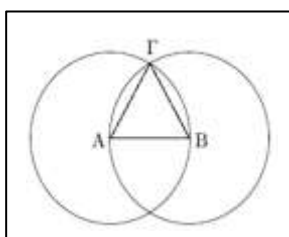


Figure of Proposition 1_1.

Proof procedure

- Assumptions-

- a1. Let AB be the given line segment.
- a2. Draw a circle with center A and radius AB .
- a3. Draw a circle with center B and radius BA .
- a4. Let Γ be an intersection point of the two circles and connect Γ and A , and Γ and B by line segments.

Proof

P1. From a2, $AB = \Gamma A$.

P2. From a3, $AB = \Gamma B$.

P3. From a4, $\Gamma A = \Gamma B$.

From P1, P2, and P3, an isosceles triangle ΓAB is created on the line segment AB .

The implementation on Isabelle is as follows.

theorem (in areadef) Proposition1_1:

fixes $A B \Gamma :: \text{point}$

and $AB \Gamma A \Gamma B :: \text{segment}$

and $\text{Circ}AB \text{Circ}BA :: \text{circle}$

assumes

" $AB = \text{Se } A B$ " " $\Gamma A = \text{Se } \Gamma A$ " " $\Gamma B = \text{Se } \Gamma B$ "

" $\text{Circ}AB = \text{Ci } A B$ " " $\text{Circ}BA = \text{Ci } B A$ "

" $\Gamma \text{ [on] } \text{Circ}AB$ " " $\Gamma \text{ [on] } \text{Circ}BA$ "

shows " $AB \text{ [@] } \Gamma A$ " and " $AB \text{ [@] } \Gamma B$ "

and " $\Gamma B \text{ [@] } \Gamma A$ "

proof -

from assms show " $AB \text{ [@] } \Gamma A$ "

by (simp add:circle_dist1 dist_rev3)

from assms show " $AB \text{ [@] } \Gamma B$ "

by (simp add:circle_dist1 dist_rev1 dist_rev3)

from assms show " $\Gamma B \text{ [@] } \Gamma A$ "

by (simp add:circle_dist3)

qed

After each proposition is proved, its contents are added as a new function so that it can be used for subsequent proofs.

For Proposition 1, the function is as follows.

```

locale L_Proposition1_1 = areadef +
fixes L_Prop1 1 :: "point => segment => bool"
("[p1-1] _,_ ")
assumes Prop1 1 : "[[ [p1-1] pn,(Se p1 p2)]]
==> Se p1 p2 [@] Se pn p1
^ Se p1 p2 [@] Se pn p2
^ Se pn p1 [@] Se pn p2"

```

"[P1-1] C, AB" means "create an isosceles triangle CAB on the line segment AB".

In the following, the procedure of proof , functions used and types omit.

The meanings of the symbols used are as follows.

- "A [o] B": Angle A = Angle B.
- "A_B [oo] C_D": Angle A + Angle B = Angle C + Angle D.
- "A [o-o] B_C": Angle A = Angle B + Angle C.
- "A [o>] B": Angle A > Angle B
- "A_B [oo>] C_D": Angle A + Angle B > Angle C + Angle D.
- "AB [⊥] C": C is a point on the line segment AB.
- "AB [:] CD": Line segment AB and line segment CD are parallel.
- "ABC [#] DEF": Triangle ABC = Triangle DEF.
- "ABCD [◇ =] a1": The area of the quadrangle ABCD is the area a1.
- "A1 [=] a2": Area a1 = Area a2.

Proposition 1_6 (Triangle Property): If the two angles of a triangle are equal to each other, then the two corresponding sides to the same angle are also equal to each other.

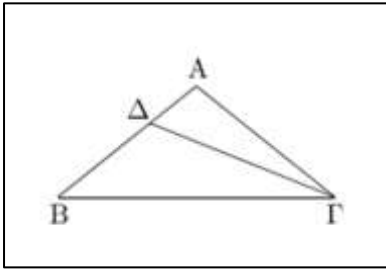


Figure of Proposition 1_

theorem (in L_Proposition1_5) Proposition1_6:

fixes $A B \Gamma \Delta :: \text{point}$

and $AB A\Gamma \Delta\Gamma B\Gamma \Gamma B B\Delta \Gamma A :: \text{segment}$

and $AB\Gamma A\Gamma B \Delta B\Gamma \Delta\Gamma B :: \text{angle}$

and $A\Gamma\Delta \Delta\Gamma B :: \text{ang_list}$

assumes N:

" $AB = \text{Se } A B$ " " $A\Gamma = \text{Se } A \Gamma$ " " $\Delta\Gamma = \text{Se } \Delta \Gamma$ "

" $BA = \text{Se } B A$ " " $B\Gamma = \text{Se } B \Gamma$ " " $\Gamma B = \text{Se } \Gamma B$ "

" $B\Delta = \text{Se } B \Delta$ " " $\Gamma A = \text{Se } \Gamma A$ "

" $AB\Gamma = \text{An } A B \Gamma$ " " $A\Gamma B = \text{An } A \Gamma B$ "

" $\Delta B\Gamma = \text{An } \Delta B \Gamma$ " " $\Delta\Gamma B = \text{An } \Delta \Gamma B$ "

" $A\Gamma\Delta = \text{An } A \Gamma \Delta$ "

" $A\Gamma\Delta \Delta\Gamma B = \text{Anl } A\Gamma\Delta \Delta\Gamma B$ "

" $\Delta B\Gamma [o] A\Gamma B$ " " $A\Gamma B [o-o] A\Gamma\Delta \Delta\Gamma B$ "

" $[p1-3] BA, A\Gamma, \Delta$ " " $AB\Gamma [o] A\Gamma B$ "

shows " $\neg\neg AB [@] A\Gamma$ "

proof

assume A1 : " $\neg AB [@] A\Gamma$ "

from N have P1 : " $B\Delta [@] A\Gamma$ "

by (simp add:Prop1_3 dist_rev1)

from N P1 have P2 : " $B\Delta [@] \Gamma A$ "

by (blast intro:dist_rev3)

from N have P3 : " $B\Gamma [@] \Gamma B$ "

by (blast intro:dist_rev2 dist_rev3)

from N P2 P3 have P4 : " $\Delta\Gamma B [o] AB\Gamma$ "

by (blast intro:Prop1_4_4 angle_rev3)

from N P4 have P5 : " $A\Gamma B [o] \Delta\Gamma B$ "

by (blast intro:angle_trans angle_rev1)

from N have P6 : " $A\Gamma B [o>] \Delta\Gamma B$ "

by (simp add:greater_ang_sum_trans1)
 from P5 P6 show False
 by (simp add:angle_contraposition1)
 qed

Proposition 1_13 (Straight Line Property): If a straight line is constructed on another straight line to make two angles, the construction will create two right angles or two angles whose sum is equal to two right angles.

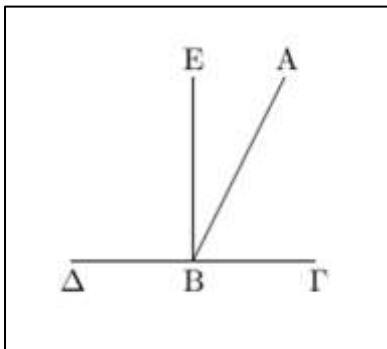


Figure of Proposition 1_13

theorem (in L_Proposition1_12) Proposition1_13:

fixes A B E Γ Δ :: point

and ΔΓ EB AB:: segment

and EBΔ EBA ΓBA ΔBA EBΓ hπ :: angle

and EBA_EBA EBΓ_EBΔ ΔBA_ΓBA EBA_ΓBA EBA_EBA_ΓBA π :: ang_list

assumes

"ΔΓ = Se Δ Γ" "EB = Se E B" "AB = Se A B"

"EBΔ = An E B Δ" "ΔBA = An Δ B A"

"EBA = An E B A" "ΓBA = An Γ B A"

"EBΓ = An E B Γ"

"EBA_EBA = Anl EBA EBA"

"EBA_ΓBA = Anl EBA ΓBA"

"EBΓ_EBΔ = Anl EBΓ EBΔ"

"ΔBA_ΓBA = Anl ΔBA ΓBA"

"EBA_EBA_ΓBA = Anlt EBA EBA ΓBA"

"Right_angle hπ" "π = Anl hπ hπ"

"ΔΓ [⊥] B"

"ΔBA [o-o] EBA_EBA" "EBΓ [o-o] EBA_ΓBA"

"[p1-11] ΔΓ,EB"

shows "ΔBA [o] ΓBA ==>

Right_angle $\Delta BA \wedge$ Right_angle ΓBA "
 and " $\Delta BA_{\Gamma BA}$ [oo] π "

proof -

from assms show " ΔBA [o] $\Gamma BA \implies$
 Right_angle $\Delta BA \wedge$ Right_angle ΓBA "
 by (simp add:Right_angle_def1)
 from assms have P1 : "Right_angle $EB\Gamma$ "
 by (simp add:Prop1_11)
 from assms have P2 : "Right_angle $EB\Delta$ "
 by (simp add:Prop1_11)
 from assms P1 have P3 : " $EB\Gamma$ [o] $h\pi$ "
 by (simp add:Right_angle_def3)
 from assms P2 have P4 : " $EB\Delta$ [o] $h\pi$ "
 by (simp add:Right_angle_def3)
 from assms P3 P4 have P5 : " $EB\Gamma_{EB\Delta}$ [oo] π "
 by (simp add:ang_list_ang5)
 from assms have P6 :
 " $\Delta BA_{\Gamma BA}$ [oo] $EB\Delta_{EBA_{\Gamma BA}}$ "
 by (simp add:ang_sum_anlt1)
 from assms have P7 :
 " $EB\Gamma_{EB\Delta}$ [oo] $EB\Delta_{EBA_{\Gamma BA}}$ "
 by (simp add:ang_sum_anlt2)
 from assms P6 P7 have P8 :
 " $\Delta BA_{\Gamma BA}$ [oo] $EB\Gamma_{EB\Delta}$ "
 by (blast intro:ang_list_trans ang_list_rev1)
 from assms P5 P8 show " $\Delta BA_{\Gamma BA}$ [oo] π "
 by (blast intro:ang_list_trans)

qed

Proposition 1_29 (Parallel Property): The opposite angles formed by a straight line intersecting two parallel lines are equal to each other, i.e., the outer angle is equal to the internal angle, and the sum of the ipsilateral internal angles is equal to two right angles.

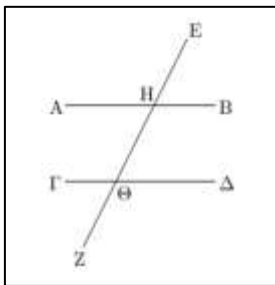


Figure of Proposition 1_29

theorem (in L_Proposition1_28) Proposition1_29:

fixes A B Γ Δ E Z H Θ :: point
 and AB ΓΔ EZ EΘ BA :: segment
 and AHΘ HΘΔ EHB BHΘ hπ:: angle
 and BHΘ_HΘΔ BHΘ_AHΘ BHΘ_EHB
 π :: ang_list

assumes N :

"AB = Se A B" "ΓΔ = Se Γ Δ" "EZ = Se E Z"
 "EΘ = Se E Θ" "BA = Se B A"
 "EHB = An E H B" "HΘΔ = An H Θ Δ"
 "BHΘ = An B H Θ" "AHΘ = An A H Θ"
 "BHΘ_HΘΔ = Anl BHΘ HΘΔ"
 "BHΘ_AHΘ = Anl BHΘ AHΘ"
 "BHΘ_EHB = Anl BHΘ EHB"
 "AB [⊥] H" "ΓΔ [⊥] Θ" "EΘ [⊥] H"
 "BA [⊥] H"
 "Right_angle hπ" "π = Anl hπ hπ"
 "BHΘ_AHΘ [oo] π" "BHΘ_EHB [oo] π"
 "AB [:] ΓΔ"

shows "¬¬ AHΘ [o] HΘΔ" and "EHB [o] HΘΔ"
 and "BHΘ_HΘΔ [oo] π"

proof -

have S1 : "¬¬ AHΘ [o] HΘΔ"

proof assume A : "¬ AHΘ [o] HΘΔ"

from N A have P1 : "AHΘ [o>] HΘΔ"

by (simp add:angle_contraposition3)

from N P1 have P2 :

"BHΘ_AHΘ [oo>] BHΘ_HΘΔ"

by (simp add:greater_ang_list1)

```

from N P2 have P3 : "π [oo>] BHΘ_HΘΔ"
  by (simp add:greater_ang_list_trans1)
from N P3 have P4 : "¬ AB [:] ΓΔ"
  by (simp add:postulate1_5)
from N P4 show False by simp
qed
from N have P5 : "EHB [o] AHΘ"
  by (simp add:Prop1_15 angle_rev2)
from S1 show "¬ AHΘ [o] HΘΔ" by simp
from N S1 P5 show S2 : "EHB [o] HΘΔ"
  by (blast intro:angle_trans)
from N S2 have P6 : "BHΘ_EHB [oo] BHΘ_HΘΔ"
  by (simp add:ang_list_ang2)
from N P6 show "BHΘ_HΘΔ [oo] π"
  by (blast intro:ang_list_trans ang_list_rev1)
qed

```

Proposition 1_41 (Area): If a parallelogram has the same base as a triangle and is between the same parallel lines, then the parallelogram is twice the triangle in area.

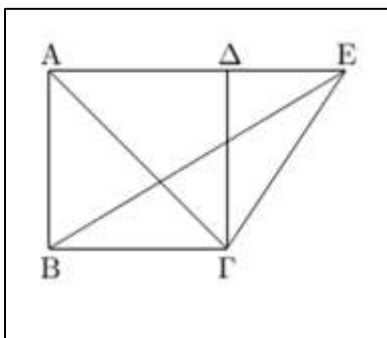


Figure of Proposition 1_41

theorem (in L_Proposition1_40) Proposition1_41:

fixes A B Γ Δ E :: point

and BΓ AE AΔ AB ΔΓ :: segment

and TriEBΓ TriABΓ :: triangle

and ABΓΔ :: quadrangle

assumes

"BΓ = Se B Γ" "AE = Se A E" "AΔ = Se A Δ"

"AB = Se A B" "ΔΓ = Se Δ Γ"

"TriEB Γ = Tri E B Γ " "TriAB Γ = Tri A B Γ "

"AB $\Gamma\Delta$ = Qu A B $\Gamma \Delta$ "

"B Γ [:] AE" "A Δ [:] B Γ " "AB [:] $\Delta\Gamma$ "

shows

"AB $\Gamma\Delta$ [\diamond =] Cult TriEB Γ add TriEB Γ "

proof -

from assms have P1 : "TriAB Γ [#] TriEB Γ "

by (simp add:Prop1_37)

from assms have P2 : "AB $\Gamma\Delta$ [\diamond =]

Cult TriAB Γ add TriAB Γ "

by (simp add:Prop1_34_3_4)

from assms P1 have P3 :

"Cult TriAB Γ add TriAB Γ [=]

Cult TriEB Γ add TriEB Γ "

by (simp add:calcdef_tri2)

from assms P2 P3 show "AB $\Gamma\Delta$ [\diamond =]

Cult TriEB Γ add TriEB Γ "

by (simp add:quar_trans1)

qed

Acknowledgment

This work was supported in part by MEXT, Japan.

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